

Logic.

The fact that $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function is sometimes written

$$\forall x_0 \forall \varepsilon \exists \delta \forall x ((\varepsilon > 0) \wedge ((0 < |x - x_0| < \delta) \rightarrow (|f(x) - f(x_0)| < \varepsilon)))$$

We will learn how to read and write such sentences, and how to test whether a sentence like this is true.

We will discuss the following four topics.

- **Syntax.** (What are the accepted ways to combine symbols into sensible expressions?)
- **Meaning.** (What does it mean for an expression to be true or false?)
- **Proof.** (How can we determine truth or communicate it to others?)
- **The relationship between truth and provability.** (Are all provable statements true? Are all true statements provable?)

(I) Syntax.

(1) An alphabet of symbols.

- (a) variables: x_1, x_2, x_3, \dots
- (b) logical symbols
 - (i) equality: $=$
 - (ii) logical connectives: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$.
 - (iii) quantifiers: \exists, \forall
- (c) nonlogical symbols: operation symbols, predicate symbols
- (d) punctuation: parentheses, commas.

(2) Formulas and sentences.

(II) Meaning.

- (1) The logical connectives.
- (2) The equivalence of $(H \rightarrow C)$, $((\neg C) \rightarrow (\neg H))$ and $((H \wedge \neg C) \rightarrow \text{False})$.
- (3) Structures.
- (4) Valuations.
- (5) The symbol \models
- (6) The equivalence of $\neg(\exists x P)$ and $\forall x(\neg P)$.

(III) Proof.

- (1) Axioms.
- (2) Rules of deduction.
- (3) The symbol \vdash

(IV) The relationship between truth and provability.