

Formulas for Distributions.

How many ways can one distribute k balls to n distinct boxes?

Assumption 1. No bound on the number of balls per box.

Each box must get a ball?

		Yes	No
Balls distinct?	Yes	$n!S(k, n)$	n^k
	No	$\left(\left(\begin{smallmatrix} n \\ k-n \end{smallmatrix}\right)\right)$	$\left(\left(\begin{smallmatrix} n \\ k \end{smallmatrix}\right)\right)$

Assumption 2. Each box gets at most one ball (so $k \leq n$).

Each box must get a ball?

		Yes (so $k = n$)	No
Balls distinct?	Yes	$n!$	$(n)_k$
	No	1	$\left(\begin{smallmatrix} n \\ k \end{smallmatrix}\right)$

- (1) How many ways are there to distribute 12 different books to 3 people? What if each person must get at least one book?
- (2) How many ways are there to distribute 12 identical textbooks to three shelves? How many ways to distribute 12 different books to three shelves?
- (3) How many 5 digit numbers have their digits in increasing or decreasing order? How many have their digits in nondecreasing or nonincreasing order? (If $n = abcde$, then the digits are in increasing order if $a < b < c < d < e$ and in nondecreasing order if $a \leq b \leq c \leq d \leq e$.)
- (4) How many positive integral solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 100$? How many nonnegative integral solutions are there?
- (5) How many ways are there to make 3 fruit baskets from 8 pineapples, 10 pomegranates, 6 coconuts and 20 figs if each basket must contain each kind of fruit?