DISCRETE MATH (MATH 2001)

REVIEW SHEET I

I. Set Theory
   (a) Informal notion of a set. The axioms.
   (b) Valid constructions of new sets (pairing, union, power set, separation, intersection)
   (c) Empty set, successor of a set.
   (d) Inductive sets, natural numbers.
   (e) Recursive definitions of $x + y, x \cdot y$ and x^y for $x, y \in \mathbb{N}$.
   (f) Russell's Paradox.

II. Logic
   (a) Formulas
      (i) Symbols: variables, equality, logical connectives, quantifiers, predicate symbols, punctuation symbols.
      (ii) Atomic formulas, formulas and sentences.
      (iii) Formula trees.
   (b) Propositional logic
      (i) Truth tables.
      (ii) Tautologies, contradictions, logical equivalence.
   (c) Structures (definition and examples).
   (d) Truth of a sentence in a structure.
      (i) Converting a sentence to prenex form.
      (ii) Quantifier games to determine the truth of a sentence in prenex form in a given structure.
      (iii) The meaning of $\Sigma \models \beta$.
   (e) Proof. (Part 1.)
      (i) Definition of a formal proof.
      (ii) Modus ponens, modus tollens.

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.
(1) Know the definitions of new concepts, and the meanings of the definitions.
(2) Know the statements and meanings of the axioms and major theorems.
(3) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(4) Know how to perform the different kinds of calculations discussed in class.
(5) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(6) Know how to correct mistakes made on old HW.

More specific advice.

Be prepared to demonstrate understanding in the following ways.

(1) Know the definitions of the following: empty set; successor; subset; power set; union; intersection; unordered pair; ordered pair; inductive set; natural numbers; arithmetic operations on $\mathbb{N}$; logical connectives; propositional formula; truth table; tautology; contradiction; logical equivalence; predicate; atomic formulas; formula; formula tree; scope of a quantifier; free variable; bound variable; sentence; prenex form.

(2) Know the statements and meanings of: the axiom of the empty set; the axiom of extensionality; the axiom of infinity; the axiom of pairing; the axiom of union; the axiom of power set; the axiom of separation; Russell's Paradox.

(4) Know how to: prove two sets are equal; rewrite English sentences as formal sentences and rewrite formal sentences as English sentences; test if two propositions are logically equivalent or whether one proposition is a tautology or contradiction; create a formula tree; put a formula in prenex form; test a sentence for truth in a structure.

Test your understanding.

(1) Explain why $2 + 2 = 4$.

(2) Define the natural numbers.

(3) Show that $A \subseteq B$ if and only if $A \cup B = B$.

(4) Prove that $2^{1+2} = 2^1 2^2$.

(5) Write a formal sentence expressing the axiom of union. Then draw a formula tree for your sentence.

(6) Show that $(A \to (\forall x \;\; B(x))) \;\; \equiv \;\; \forall x \;\; (A \to B(x))$ if x is not free in A and $((\forall x \; A(x)) \to B) \; \equiv \; \exists x \; (A(x) \to B)$ if x is not free in B.

(7) Put the axiom of extensionality, $\forall A \forall B((A = B) \leftrightarrow (\forall x((x \in A) \leftrightarrow (x \in B))))$, in prenex form.

(8) Describe a winning strategy for either $\exists$ or $\forall$ which determines the truth of

$$\forall x \; (\exists y \; (x = y^2) \to \exists z \; (x + 1 = z^2))$$

in (i) $\langle \mathbb{R}; +, \cdot, 1 \rangle$, (ii) $\langle \mathbb{Z}; +, \cdot, 1 \rangle$.