DISCRETE MATH
QUIZ 7

Name:______________________________

You have 10 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

1. Show that $m \cdot (n + k) = (m \cdot n) + (m \cdot k)$ for all $m, n, k \in \mathbb{N}$.

Let's prove the statement "$m \cdot (n + k) = (m \cdot n) + (m \cdot k)$ for all $m, n \in \mathbb{N}$" by induction on k.

Basis of induction, $(k = 0)$.
Must show $m \cdot (n + 0) = (m \cdot n) + (m \cdot 0)$.

$$
\begin{aligned}
m \cdot (n + 0) &= m \cdot n && (\text{IC}, +) \\
&= (m \cdot n) + 0 && (\text{IC}, +) \\
&= (m \cdot n) + (m \cdot 0) && (\text{IC}, \cdot)
\end{aligned}
$$

Inductive step.
Assume $m \cdot (n + k) = (m \cdot n) + (m \cdot k)$ and show $m \cdot (n + S(k)) = (m \cdot n) + (m \cdot S(k))$.

$$
\begin{aligned}
m \cdot (n + S(k)) &= m \cdot S(n + k) && (\text{RR}, +) \\
&= (m \cdot (n + k)) + m && (\text{RR}, \cdot) \\
&= ((m \cdot n) + (m \cdot k)) + m && (\text{Inductive Hypothesis}) \\
&= (m \cdot n) + ((m \cdot k) + m) && (\text{Associative Law for } +) \\
&= (m \cdot n) + (m \cdot S(k)) && (\text{RR}, \cdot)
\end{aligned}
$$