

DISCRETE MATH

QUIZ 4

Name: _____

You have 10 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

1. Write $((0 < 1) \leftrightarrow (\forall x(0 < x)))$ in prenex form.

$$\begin{aligned}
 ((0 < 1) \leftrightarrow (\forall x(0 < x))) &\equiv ((0 < 1) \rightarrow (\forall x(0 < x))) \wedge ((\forall x(0 < x)) \rightarrow (0 < 1)) \\
 &\equiv ((\neg(0 < 1)) \vee (\forall x(0 < x))) \wedge ((\neg(\forall x(0 < x)) \vee (0 < 1))) \\
 &\equiv ((\neg(0 < 1)) \vee (\forall x(0 < x))) \wedge ((\exists x(\neg(0 < x))) \vee (0 < 1)) \\
 &\equiv ((\neg(0 < 1)) \vee (\forall x(0 < x))) \wedge ((\exists y(\neg(0 < y))) \vee (0 < 1)) \\
 &\equiv \forall x \exists y ((\neg(0 < 1)) \vee (0 < x)) \wedge ((\neg(0 < y)) \vee (0 < 1))
 \end{aligned}$$

2. Determine whether

$$\forall w \exists x \forall y \exists z ((w - x)(y - z) = 1)$$

is true in $(\mathbb{R}, \{+, -, 0, \cdot, 1\})$ by giving a winning strategy for the appropriate quantifier.

The sentence is true in \mathbb{R} . A winning strategy for \exists is to choose $x = w - 1$ and $z = y - 1$. Then $(w - x)(y - z) = 1 \cdot 1 = 1$, as desired.