

Practice Problems.

1. Give an indirect proof of “There is a unique set with no elements.” Then explain why statements of the form “There is a unique X with property Y ” should be easier to prove indirectly than directly.

2.

(a) Prove or disprove: “If f is increasing and g is increasing, then $f \circ g$ is increasing.”

(b) Prove or disprove: “If f is decreasing and g is decreasing, then $f \circ g$ is decreasing.”

3. Prove or disprove: “If f is bounded above and g is bounded above, then $f + g$ is bounded above.”

4. Give examples of statements of the form $H \rightarrow C$ such that the following properties hold. If no example exists, explain why.

- (a) The statement and its converse are both true.
- (b) The statement is true, but the converse is false.
- (c) The statement is false, but the converse is true.
- (d) The statement and its converse are both false.
- (e) The statement is true and its contrapositive is false.
- (f) The contrapositive and the converse are both false.
- (g) The contrapositive is true and the converse is false.