

Practice Problems.

1. Write the following statements about a function $f: \mathbb{R} \rightarrow \mathbb{R}$ as formal sentences in prenex form.

- (a) f is an injective (= 1-1) function.
- (b) f is an surjective (= onto) function.
- (c) f is strictly increasing. (I.e., f preserves $<$.)
- (d) f is nondecreasing. (I.e., f preserves \leq .)
- (e) f is bounded. (I.e., there is a real number B such that $f(x) \leq B$ for all x .)
- (f) f is bounded on the positive real numbers.
- (g) f is continuous at every real number.

2. Write the following as formal sentences in prenex form.

- (a) The Axiom of Infinity. (=There is an inductive set.)
- (b) There exists arbitrarily large real numbers that are zeros of $\sin(x)$.
- (c) Every quadratic polynomial has at most 2 roots. (Hint: look at 3(b).)
- (d) $((\forall x P(x)) \leftrightarrow Q)$

3. Which of the following sentences are true in $(\mathbb{R}, \{+, -, 0, \cdot, 1, <\})$?

(a)

$$\forall y_1 \exists x (x + y_1 = 0).$$

(b)

$$\forall y_1 \forall y_2 \exists x (x^2 + y_1 x + y_2 = 0).$$

(c) (Hard.)

$$\forall y_1 \forall y_2 \forall y_3 \exists x (x^3 + y_1 x^2 + y_2 x + y_3 = 0).$$

4. Explain whether or not the following functions have a limit at the given point.

(a) $f(x) = |x|$ at $x = 1$. At $x = 0$?

(b) $f(x) = x^2$ at $x = 1$. At $x = 0$?

(c) (Hard!) $f(x) = \sin(\frac{1}{x})$ at $x = 0$.