Practice Problems About Russell's Paradox.

(1) Find a sequence of sets satisfying $\cdots A_3 \subsetneq A_2 \subsetneq A_1 \subsetneq A_0$.

(2) Find a sequence of sets satisfying $B_0 \in B_1 \in B_2 \in B_3 \in \cdots$.

(3) (Russell's pair of ducks.) Bertrand Russell gives a pair of ducks to exactly those who do not give a pair of ducks to themselves. Show that
   (a) Russell cannot fail to give himself a pair of ducks.
   (b) It is not possible for Russell to give himself a pair of ducks.

(4) Show that the union of all sets is not a set.

(5) Find all 3-element sets A such that $A \subseteq \mathcal{P}(A)$. (There are only 2 of them.)

The Axioms of Set Theory.

Equality

(1) (Extensionality) Two sets are equal if they have the same elements.

Existence of Special Sets

(2) (Empty Set) There is a set with no elements.
(3) (Infinity) There is an inductive set.

Creation of New Sets

(4) (Pairing) If A and B are sets, then $\{A, B\}$ is a set.
(5) (Union) If I is a set, and A_i is a set for each $i \in I$, then $\bigcup_{i \in I} A_i$ is a set.
(6) (Power Set) If A is a set, then $\mathcal{P}(A)$ is a set.
(7) (Separation) If A is a set and P is a property given by a formula, then $\{x \in A \mid P(x)\}$ is a set.
(8) (Replacement) If A is a set and F is a function given by a formula, then $\{F(x) \mid x \in A\}$ is a set.
(9) (Choice) If $\{A_i \mid i \in I\}$ is set of nonempty pairwise disjoint sets, then there is a set C such that $|A_i \cap C| = 1$ for every i.

Sets have Special Properties

(10) (Regularity) If A is a nonempty set, then there is an $x \in A$ such that x and A are disjoint.