

## Practice Problems Involving Induction.

1. Assume that the derivative of  $x$  has been shown to 1. Use the product rule for differentiation and induction to show that  $[x^n]' = nx^{n-1}$ .
2. Show by induction that if  $A$  has  $n$  elements, then the power set  $\mathcal{P}(A)$  has  $2^n$  elements.
3. Suppose that  $n$  lines are drawn in the plane so that no 2 lines are parallel and no 3 lines pass through the same point. Show that the lines divide the plane into  $\frac{n^2 + n + 2}{2}$  regions.
4. There are  $n$  points in space where some pairs of points are joined by line segments. Suppose that the whole configuration of points and segments is connected, but if one segment is deleted then it will no longer be connected. Show that there must be  $n - 1$  line segments.
5. The Towers of Hanoi puzzle has three vertical rods on a board and a series of rings of different radii on the first rod. The rings are stacked so that no ring is on top of a smaller ring. The object of the puzzle is to move all rings to the third rod by moving them one at a time to other rods, while never placing any ring on top of a smaller ring. Show that this puzzle with  $n$  rings can be solved for any value of  $n$ .