DISCRETE MATH (MATH 2001)

REVIEW SHEET I

I. Set Theory
  (a) Informal notion of a set. The axioms.
  (b) Valid constructions of new sets (pairing, union, power set, comprehension, intersection)
  (c) Empty set, successor of a set.
  (d) Inductive sets, natural numbers.
  (e) Russell's Paradox.

II. Induction
  (a) Ordinary induction.
  (b) Strong induction.
  (c) Recursive definitions of arithmetic operations on $\mathbb{N}$: $x + y, xy, x^y$.
  (d) Use of induction to prove laws of arithmetic.

III. Logic
  (a) Formulas
    (i) Alphabet of symbols: variables, equality, connectives, quantifiers, predicate symbols, punctuation symbols.
    (ii) Terms, atomic formulas, formulas and sentences.
    (iii) Formula trees, term trees.
  (b) Propositional logic
    (i) Truth tables.
    (ii) Tautologies, contradictions, logical equivalence.
    (iii) Contrapositive and converse.
    (iv) Equivalence of $(H \to C)$, $((\neg C) \to (\neg H))$, and $((H \wedge (\neg C)) \to \text{False})$. Methods of proof.
    (v) Disjunctive normal form.
  (c) Structures (definition and examples).
  (d) Truth of a sentence in a structure.
    (i) Converting a sentence to prenex form (including: scope of a quantifier, free and bound variables, rules for changing the order of quantifiers and connectives).
    (ii) Quantifier games to determine the truth of a sentence in prenex form in a given structure.

IV. Relations
  (a) Ordered pairs, triples and n-tuples. $A \times B$.
  (b) Definition of a function. Representations of functions. Composition.
  (c) Injections, surjections, bijections. Canonical factorization of a function.

(d) Kernel of a function. Equivalence relations. Partitions. Relationships between these three.
(e) Posets: strict and nonstrict orders. Extensions. Linear orders.

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.

Sample Problems.

(1) Explain why $2 + 2 = 4$.
(2) Show that $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.
(3) Explain why induction is a valid form of proof. (Your explanation should make use of the fact that $\mathbb{N}$ is a subset of every inductive set.)
(4) Prove that $m(n + k) = (mn) + (mk)$ for all $m, n, k \in \mathbb{N}$.
(5) Write a formal sentence expressing the axiom of union. Then draw a formula tree for your sentence.
(6) Put $((p \to q) \to r)$ in disjunctive normal form.
(7) Describe a winning strategy for either $\exists$ or $\forall$, which determines the truth of

$$\forall x\, \forall y\, \exists z\, ((x < y) \to ((x < z) \wedge (z < y)))$$

in (i) $\langle \mathbb{R}; < \rangle$, (ii) $\langle Z; < \rangle$.
(8) Put $(A \leftrightarrow \forall x\, B(x))$ in prenex form. You may assume that A has no free variables.
(9) What is a function? (Give the definition.)
(10) What is the kernel of the squaring function $F\colon \mathbb{R} \to \mathbb{R}\colon x \mapsto x^2$?
(11) How many different partitions are there on the set $\{1, 2, 3, 4\}$?
(12) Draw an example of an ordered set that has 1 minimum element and 5 maximal elements. Then draw one that has 0 minimum elements, 1 minimal element and 5 maximal elements.