DISCRETE MATHEMATICS
HOMEWORK ASSIGNMENT I

PROBLEMS

1. Consider the following two properties: (a) $A \in B$, and (b) $A \subseteq B$.

(i) Give examples of sets A and B where property (a) holds but (b) fails.

(ii) Give examples of sets A and B where property (a) fails but (b) holds.

(iii) Give examples of sets A and B where properties (a) and (b) both hold.

(iv) Give examples of sets A and B where properties (a) and (b) both fail.

(v) Explain why it is that if A and B are distinct natural numbers, then both (a) and (b) hold or both (a) and (b) fail.

2. A set T is *transitive* if $R \in S \in T$ implies $R \in T$.

(i) Give an example of a set that is transitive.

(ii) Give an example of a set that is not transitive.

(iii) Show that A is transitive iff $A \subseteq \mathcal{P}(A)$.

3. (Ordered pairs) If A and B are sets, let $(A, B) := \{\{A\}, \{A, B\}\}$. The set (A, B) is called an *ordered pair*. Explain why

(i) (A, B) is a legitimate set. (That is, show that the axioms of set theory guarantee that if A and B are sets, then so is (A, B).)

(ii) $(A, B) = (C, D)$ iff $A = C$ and $B = D$.