

The Axioms of Set Theory.

Equality

- (1) (Extentionality) Two sets are equal if they have the same elements.

Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.
(Call it the emptyset and denote it by $\emptyset, \{ \}$ or 0.)
- (3) (Infinity) There is an inductive set.
(Equivalently, there is a set containing $\mathbb{N} = \{0, 1, 2, \dots\}$.)

Creation of New Sets

- (4) (Pairing) If A and B are sets, then $\{A, B\}$ is a set.
- (5) (Union) If I is a set, and A_i is a set for each $i \in I$, then $\bigcup_{i \in I} A_i$ is a set.
- (6) (Power Set) If A is a set, then $\mathcal{P}(A)$ is a set.
- (7) (Comprehension) If A is a set and P is a property given by a formula, then $\{x \in A \mid P(x)\}$ is a set.
- (8) (Replacement) If A is a set and F is a function given by a formula, then $\{F(x) \mid x \in A\}$ is a set.
- (9) (Choice) If $\{A_i \mid i \in I\}$ is set of nonempty disjoint sets, then there is a set C such that $|A_i \cap C| = 1$ for every i .

Sets have Special Properties

- (10) (Foundation) There is no infinite sequence of sets $\dots A_3 \in A_2 \in A_1 \in A_0$.