1.7 (cont.) an example of a ring with a nil ideal I satisfying $I = I^2$.

the groupring $\mathbb{F}_2[\mathbb{Z}_{2^\infty}]$ is such a ring where $\mathbb{F}_2$ is the two element field &

$$\mathbb{Z}_{2^\infty} = \{\exp(2\pi i n/2^m) \mid n \,\&\, m \in \mathbb{Z}^+\}$$

(prüfer group)

we take the ideal I generated by the subset

$$\{g-1 \mid g \in \mathbb{Z}_{2^\infty}\}$$

<u>$I = I^2$</u> we have $g = \exp(2\pi i n/2^m) = (\exp(2\pi i n/2^{m+1}))^2 = h^2$

so that $(h-1)^2 = h^2 - 2\cdot h + 1 = h^2 - 1 = g - 1$

& $g - 1 \in I^2$. Hence $I \subseteq I^2$ & so $I = I^2$

<u>I is nil</u> with g as defined above,

$$(g-1)^{2^m} = \sum_{i=0}^{2^m} \binom{2^m}{i} g^{2^m - i} = g^{2^m} - 1 = 1 - 1 = 0$$

which is what we needed to show