

Logic. Chapters 5 and 6.

The fact that $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function is sometimes written

$$\forall x_0 \forall \varepsilon \exists \delta \forall x ((\varepsilon \leq 0) \vee ((0 < |x - x_0| < \delta) \rightarrow (|f(x) - f(x_0)| < \varepsilon)))$$

Part of what we will learn about logic is how to read and write such sentences.

We will discuss the following four topics.

- **Syntax.** (What are the accepted ways to combine symbols into sensible expressions?)
- **Meaning.** (What does it mean for an expression to be true or false?)
- **Proof.** (How can we determine truth or communicate it to others?)
- **The relationship between truth and provability.** (Are all provable statements true? Are all true statements provable?)

(I) Syntax.

- (1) An alphabet of symbols.
 - (a) variables: x_1, x_2, x_3, \dots
 - (b) logical symbols
 - (i) equality: $=$
 - (ii) logical connectives: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$.
 - (iii) quantifiers: \exists, \forall
 - (c) nonlogical symbols: operation symbols, predicate symbols
 - (d) punctuation: parentheses, comma.
- (2) Formulas and sentences.
- (3) The equivalence of $(H \rightarrow C)$, $((\neg C) \rightarrow (\neg H))$ and $((H \wedge \neg C) \rightarrow \text{False})$.

(II) Meaning.

- (1) Structures.
- (2) Valuations.
- (3) The symbol \models
- (4) The equivalence of $\neg(\exists x P)$ and $\forall x(\neg P)$.

(III) Proof.

- (1) Axioms.
- (2) Rules of deduction.
- (3) The symbol \vdash

(IV) The relationship between truth and provability.

Exercises.

- (1) Write formally “ E is an equivalence relation” and “ \leq is a linear order”.
- (2) Write the first seven axioms of set theory formally.
- (3) Write formally the three statements (i) “ $f'(1) = 2$ ”, (ii) “ f is differentiable on $(0, 1)$ ”, and (iii) “ $f'(x) = g(x)$ on $(0, 1)$ ”.
- (4) Write the following theorems of calculus formally: The Intermediate Value Theorem, The Extreme Value Theorem, The Mean Value Theorem.