

## Formulas for Distributions.

How many ways can one distribute  $k$  balls to  $n$  distinct boxes?

Assumption 1. No bound on the number of balls per box.

Each box must get a ball?

		Yes	No
Balls distinct?	Yes	$n!S(k, n)$	$n^k$
	No	$\binom{k-1}{n-1}$	$\binom{n+k-1}{n-1}$

Assumption 2. Each box gets at most one ball (so  $k \leq n$ ).

Each box must get a ball?

		Yes (so $k = n$ )	No
Balls distinct?	Yes	$n!$	$(n)_k$
	No	1	$\binom{n}{k}$

- (1) How many ways are there to distribute 12 different books to 3 people? What if each person must get at least one book?
- (2) How many ways are there to distribute 12 identical textbooks to three shelves? How many ways to distribute 12 different books to three shelves?
- (3) How many 5 digit numbers have their digits in increasing or decreasing order? How many have their digits in nondecreasing or nonincreasing order? (If  $n = abcde$ , then the digits are in increasing order if  $a < b < c < d < e$  and in nondecreasing order if  $a \leq b \leq c \leq d \leq e$ .)
- (4) How many positive integral solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 100$ ? How many nonnegative integral solutions are there?
- (5) How many ways are there to make 3 fruit baskets from 8 pineapples, 10 pomegranates, 6 coconuts and 20 figs if each basket must contain each kind of fruit?