DISCRETE MATH (MATH 2100)

SUMMARY OF TOPICS FROM 8/25/08-9/26/08

I. Set Theory (Notes, Sections 1.3-1.4)
   (a) Informal notion of a set. The axioms.
   (b) Valid constructions of new sets (pairing, union, power set, separation, intersection)
   (c) Empty set, successor of a set, natural numbers.
   (d) Inductive sets.
   (e) Russell's Paradox.
   (f) Cantor's Theorem (statement only).

II. Induction (Section 1.2)
   (a) Ordinary induction.
   (b) Strong induction.

III. Functions (Section 3.1)
   (a) Ordered pairs, $A \times B$, definition of a function, definition of a sequence.
   (b) Injections, surjections, bijections.

IV. Recursion (Section 1.5)
   (a) Recursively defined functions. Recursion Theorem (statement only).
   (b) Fibonacci numbers.
   (c) Recursive definitions of arithmetic operations on $\mathbb{N}$: $x + y, xy, x^y$.

IV. Cardinality (Section 3.2)
   (a) Definitions of $|A| = |B|$, $|A| \leq |B|$, $|A| = m$, etc.
   (b) Cantor-Schröder-Bernstein Theorem (statement only).
   (c) Sum Rule, Product Rule.

V. Binomial coefficients (Section 1.1)
   (a) Definition.
   (b) Binomial Theorem.
   (c) Counting Arguments.

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.

Sample Problems.

(1) What is a function?

(2) Show from the definitions that $n \leq 2n$,

(3) Show that any sequence of integers satisfying the recurrence

$$a_{n+1} = a_n - a_{n-1} + \cdots - a_{n-99} + a_{n-100}$$

is bounded.

(4) Explain why induction is a valid form of proof. (Your explanation should make use of the fact that $\mathbb{N}$ is the smallest inductive set.)

(5) Show that every positive integer is a sum of distinct Fibonacci numbers.