

DISCRETE MATHEMATICS

HOMEWORK ASSIGNMENT I

(Turn in underlined problems.)

Read Sections 1.3 and 1.4.

PROBLEMS

1. Consider the following two properties: (a) $A \in B$, and (b) $A \subseteq B$.

- (i) Give examples of sets A and B where property (a) holds but (b) fails.
- (ii) Give examples of sets A and B where property (a) fails but (b) holds.
- (iii) Give examples of sets A and B where properties (a) and (b) both hold.
- (iv) Give examples of sets A and B where properties (a) and (b) both fail.
- (v) Explain why it is that if A and B are distinct natural numbers, then both (a) and (b) hold or both (a) and (b) fail.

2. A set T is *transitive* if $R \in S \in T$ implies $R \in T$.

- (i) Give an example of a set that is transitive.
- (ii) Give an example of a set that is not transitive.
- (iii) Show that A is transitive iff $A \subseteq \mathcal{P}(A)$.

3. Find all transitive sets that have four elements.

4. (Ordered pairs) If A and B are sets, let $(A, B) := \{\{A\}, \{A, B\}\}$. The set (A, B) is called an *ordered pair*. Explain why

- (i) (A, B) is a legitimate set. (That is, show that the axioms of set theory guarantee that if A and B are sets, then so is (A, B) .)
- (ii) $(A, B) = (C, D)$ iff $A = C$ and $B = D$.