ALGEBRA 1
HOMEWORK ASSIGNMENT VIII

(Turn in underlined problems.)

Read over Chapters 4 and 5 (we discussed this already).

SECTION	**PROBLEMS**
4.2	**8, 9**
4.3	**13 (what about 3 classes?), 20**
4.4	**1, 18, 19**
4.5	**24, 29, 38**
5.2	**1**
5.5	**1, 6, 14, 18**

ADDITIONAL PROBLEMS

1. The action of G on itself by left multiplication induces an action of G on the subset lattice of G. Show that the G-set $\langle \mathrm{Sub}(G); G\rangle$ contains orbits of every possible isomorphism type.

2. Show that if G is a finitely generated group and H is a subgroup of finite index, then H is a finitely generated group. (Hint: Prove that, after a change of language, $\langle G/H; G\rangle$ is a finitely presentable algebra in a finite language. Then use an earlier HW problem.)

3. Show that the inner automorphism group of G cannot be a cyclic group of order greater than 1.

4. Show that any p-subgroup Q of a finite group is contained in a Sylow p-subgroup. (Hint: let Q act on the set of Sylow p-subgroups by conjugation. Show that Q must fix some Sylow P, and that $Q \leq P$.)

5. Show that every group G satisfying $|G| \leq 35$ has a normal Sylow subgroup, except S_4. (Hints: Assume that G has no normal Sylow subgroup,

and conclude that $|G| \neq p^k, pq, p^2q, pqr$ where p, q, r are distinct primes. Conclude that $|G| = 24$, and that G has four Sylow 3-subgroups. The conjugation action of G on these subgroups induces a homomorphism $\rho\colon G \to S_4$ whose image contains all 3-cycles, hence $A_4 \leq \rho(G) \leq S_4$. Eliminate the possibility that $\rho(G) = A_4$.)

6. A theorem of Zassenhaus states that a finite group G has cyclic Sylow subgroups iff $G \cong \mathbb{Z}_m \rtimes \mathbb{Z}_n$ for some relatively prime m and n. Use this theorem to determine all integers k with the property that there exists only one isomorphism type of group of size k.