ALGEBRA 1

HOMEWORK ASSIGNMENT V

(Turn in underlined problems.)

Read Sections 3.4, 3.5, Appendix II.1.

SECTION	**PROBLEMS**
3.2	**9, 14**
3.3	**4**
3.4	**8, 12**

ADDITIONAL PROBLEMS

1. If G is a simple group, what are the possibilities for the lattice $\mathrm{Norm}(G \times G)$**?**

2. Find groups $N \lhd G$, $N' \lhd G'$, with

(a) $G \cong G'$, $N \cong N'$, **but** $G/N \ncong G'/N'$.
(b) $G \cong G'$, $G/N \cong G'/N'$, **but** $N \ncong N'$.
(c) $G/N \cong G'/N'$, $N \cong N'$, **but** $G \ncong G'$.

3. Find a group G with $G \cong G \times G$.

4. (Parts (a) and (b) will be considered one problem, and parts (c) and (d) will be considered one problem.)

Let A be an abelian group of order mn where $\gcd(m, n) = 1$**. By the Chinese Remainder Theorem, there exists an integer a such that** $a \equiv 1 \pmod{m}$ **and** $a \equiv 0 \pmod{n}$**. Let** $\varphi\colon A \to A\colon x \mapsto x^a$**. Show that**

(a) φ **is an idempotent** ($\varphi(\varphi(x)) = \varphi(x)$) **endomorphism.**
(b) $A \cong \mathrm{im}(\varphi) \times \ker(\varphi)$.
(c) $|\mathrm{im}(\varphi)| = m$ **and** $|\ker(\varphi)| = n$.
(d) A **is isomorphic to a product of groups of prime power order.**

5. Show that if N is a minimal normal subgroup of a finite group G, then $N \cong S^k$ for some k and some simple group S.

6. Derive the Fundamental Theorem of Arithmetic from the Jordan-Hölder Theorem (applied to $\mathbb{Z}_n$).