ALGEBRA 1
HOMEWORK ASSIGNMENT III

(Turn in underlined problems.)

Read Sections 1.4, 1.5, 2.1, 2.3–2.5.

SECTION	**PROBLEMS**
1.4	**4**
1.6	**1, 7, 19, <u>23</u>**
2.1	**1, 2, 6, 11, 15**
2.3	**9, 16, 24, <u>26</u>**

ADDITIONAL PROBLEMS

1. An *isomorphism* between two algebras is an invertible homomorphism. (I.e., it is a homomorphism $\varphi\colon \mathbf{A} \to \mathbf{B}$ for which there is a homomorphism $\psi\colon \mathbf{B} \to \mathbf{A}$ such that $\psi \circ \varphi = \boldsymbol{id}_{\mathbf{A}}$ and $\varphi \circ \psi = \boldsymbol{id}_{\mathbf{B}}$.)

(a) Prove that a homomorphism is an isomorphism iff it is bijective. (That is, if a homomorphism is invertible as a set function, then its inverse must also be a homomorphism.)

(b) Prove that the analogous statement for ordered sets is false. (Show that the inverse of a bijective order-preserving function need not be order-preserving.)

(c) Prove that the analogous statement for graphs is false. (Show that the inverse of a bijective adjacency-preserving function need not be adjacency-preserving.)

(d) Prove that the analogous statement for topological spaces is false. (Show that the inverse of a bijective continuous function need not be continuous.)

2. Prove that subalgebra generation and congruence generation are algebraic closure operators.

3. Draw the lattice of subgroups of $\mathbb{Z}$.

4. Draw the lattice of subgroups of S_4, and identify which subgroups are normal.

5. Classify the finite groups G such that

(a) G has a nontrivial subgroup $N \leq G$ that is contained in all other nontrivial subgroups of G.

(b) G has a proper subgroup $P \leq G$ that contains all other proper subgroups of G.

Challenge Problem #3 (Prize = 1 coin)
Show that the lattice on the left is not isomorphic to the subgroup lattice of any group.

Challenge Problem #4 (Prize = same)
Show that the lattice on the right is not isomorphic to the normal subgroup lattice of any group.