University of Colorado
Department of Mathematics
Problem of the Month
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Starting with a set S of n^2 numbers, it is possible to create many different $n \times n$ matrices by placing the elements of S into the n^2 different positions of the matix. Let $\mathrm{DET}(S)$ be the set of determinants of all matrices that can be created from S in this way. For example, if $n = 2$ and $S = \{1, 2, 3, 4\}$, then

$$\begin{aligned} \mathrm{DET}(S) &= \left\{ \det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \det \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}, \ldots, \det \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}, \right\} \\ &= \{-2, -5, \ldots, 10\}. \end{aligned}$$

Problem. For a given n, how large can the set $\mathrm{DET}(S)$ be if S is a set of n^2 distinct integers?