University of Colorado Department of Mathematics Problem of the Month September 2011

Let n > 1 be an integer. (1) Show that the product $\prod_{\substack{1 \le m < n \\ \gcd(m,n)=1}} m$ is congruent to either +1 or -1 modulo n. (2) For which n is it true that $\prod_{\substack{1 \le m < n \\ \gcd(m,n)=1}} m \equiv -1 \pmod{n}$?