University of Colorado
Department of Mathematics
Problem of the Month
September 2009

Suppose that real valued functions $f(x)$, $g(x)$ and $h(x)$ are defined for all real x, and that

$$\frac{f(x) - g(y)}{x - y} = \frac{h(x) + h(y)}{2}$$

holds for all $x, y \in \mathbb{R}$ such that $x \neq y$. Show that $f = g$, f is a polynomial of degree at most 2, and $h = f'$.