University of Colorado
Department of Mathematics
Problem of the Month
December 2009-January 2010

Call a series $\sum_{n=1}^{\infty} a_n$ of positive real numbers a *descending series* if

$$a_1 \geq a_2 \geq a_3 \geq \cdots \geq 0.$$

Say that one descending series, $\sum_{n=1}^{\infty} a_n$, *dominates* another descending series, $\sum_{n=1}^{\infty} b_n$, if $a_n \geq b_n$ for all n. Say that a descending series, $\sum_{n=1}^{\infty} a_n$, *weakly dominates* a descending series, $\sum_{n=1}^{\infty} b_n$, if $a_n \geq b_n$ for infinitely many n.

Let $\sum_{n=1}^{\infty} a_n$ be a descending series. Show that the following are equivalent:

(1) Every descending series that weakly dominates $\sum_{n=1}^{\infty} a_n$ is divergent.
(2) Some multiple $c\left(\sum_{n=1}^{\infty} a_n\right) = \sum_{n=1}^{\infty} ca_n$ dominates the series $\sum_{n=1}^{\infty} \frac{1}{n}$.