University of Colorado Department of Mathematics Problem of the Month February 2015

Let a_n be a sequence of numbers defined by

$$a_{n+3} = \frac{(n^2 + n + 1)(n+1)}{n}a_{n+2} + (n^2 + n + 1)a_{n+1} - \frac{n+1}{n}a_n,$$

 $a_1 = 1, a_2 = 0, a_3 = 1$. Show that for every $n \ge 1$ a_n is an integer and, moreover, a complete square.