

Quantifiers on Number Grids

Let $\mathcal{N} = \{1, 2, 3\}$. In a grid, let

$S(a, b, c)$ mean “row a , column b contains the number c .”

A cell may contain several numbers. A blank cell contains no numbers. Every quantifier below has domain $\mathcal{N}$. Pay close attention to both the order of the quantifiers and the order of the arguments of S .

	Grid A			Grid B			Grid C		
	1	column 2	3	1	column 2	3	1	column 2	3
row 1	1	1, 2		1, 3	2	1, 2	1, 2, 3	1, 2, 3	1, 2, 3
row 2	2		2, 3	1	2	1, 2	2		1
row 3		3	1, 3	1	2	1, 2			

For each proposition and each grid, decide whether the proposition is true or false. Record your decisions in the table. Then justify them: for a true statement, explain how the required choices can be made; for a false statement, give values that show why it fails.

P1. $\forall a \exists b \exists c S(a, b, c)$

P2. $\exists a \forall b \exists c S(a, b, c)$

P3. $\forall c \exists a \forall b S(a, b, c)$

P4. $\exists b \forall c \exists a S(a, b, c)$

P5. $\forall a \exists c \forall b S(b, a, c)$

	Grid A	Grid B	Grid C
P1			
P2			
P3			
P4			
P5			

One comparison worth explaining. Choose two propositions that have different truth values on at least one grid. In words, explain what makes their quantifier requirements different.