

**Problem 1.** If  $X$  is true and  $Y$  is false, which of the following statements are true?

- A)  $X$  implies  $Y$     B)  $Y$  implies  $X$     C) Both    D) Neither

*Solution.* B)

□

**Problem 2.** If  $X$  is true and  $Y$  is false, which of the following statements are true?

- A)  $X$  or  $Y$   
B)  $X$  and  $Y$   
C)  $X$  and not  $Y$   
D) Both A) and B)  
E) Both A) and C)

*Solution.* E)

□

**Problem 3.** If  $X$  is true and  $Y$  is also true, which of the following statements are true?

- A)  $X$  or  $Y$   
B)  $X$  and  $Y$   
C)  $X$  and not  $Y$   
D) Both A) and B)  
E) Both A) and C)

*Solution.* D)

□

**Problem 4.** Which of the following mean the same as  $X \implies Y$ ?

- A) If  $X$  then  $Y$ .  
B)  $X$  only if  $Y$ .  
C)  $X$  implies  $Y$ .  
D) All of the above.  
E) A) and C).

*Solution.* D)

□

**Problem 5.** Is the statement “If  $x$  is a prime number then  $x$  is odd.” true or false?

- A) True    B) False

*Solution.* This statement is neither true nor false because it is not mathematically grammatical! The reason is that we can't assign a truth value to the sentence “ $x$  is a prime number” because it involves a variable. If  $x$  were 3 this would be a true sentence; if  $x$  were 4, it would be a false sentence.

Of course, what we mean when we write a sentence like this, what we really mean is “Every prime number is odd.” (Of course, this sentence is false.) Mathematicians do often write sentences like the one in this question because they trust you can figure out what they mean. However, in a very technical sense, this sentence is not grammatical!

The way to fix this sentence is by “binding” the variable  $x$  via a “universal quantifier”: “For every number  $x$ , if  $x$  is a prime number then  $x$  is odd.” We will learn more about this when we study quantifiers in Chapter 2.

□