Sample document with bibliography

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The following is a proof of the irrationality of the square root of 2, based on [1, p. 2, Proof of Theorem 1]:

Theorem 1. *There is no rational number r such that $r^2 = 2$.*

Proof. Suppose for the sake of contradiction that there is a rational number $r = p/q$ such that $r^2 = 2$ and p and q are both positive whole numbers. Then there is a choice of p and q where p is as small as possible. Then $p^2 = 2q^2$. But this means p^2 is even, and therefore p is also even. That means $p = 2s$ for some integer s. Hence $2q^2 = (2s)^2 = 4s^2$. But then $q^2 = 2s^2$. If we repeat the same argument again, we see that q is also even, so $q = 2t$, and therefore $(2t)^2 = 2s^2$, which implies $s^2 = 2t^2$. Then $r = p/q = (p/2)/(q/2) = s/t$. But this representation of r has a smaller numerator than p, which contradicts the assumption about p. Therefore the original assumption, that there was a rational number r such that $r^2 = 2$, must have been false. □

Another version of the same proof can be found in [2, p. 139].

References

[1] Joel David Hamkins, *Proof and the art of mathematics*.

[2] Richard Hammack, *Book of proof*, 3e. https://www.people.vcu.edu/~rhammack/BookOfProof/Main.pdf