CHECK IN 14

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Recall [Growth rates]: Suppose we have sequences $\{a_n\}$ and $\{b_n\}$ such that $a_n > 0$ and $b_n \geq 0$ for all n, and such that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n.$$

Then if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0,$$

we either say that $\{a_n\}$ **shrinks faster than** $\{b_n\}$ or equivalently, that $\{b_n\}$ **shrinks slower than** $\{a_n\}$.

The limit comparison test tells us that the growth rate of a sequence will determine the convergence or divergence of the series generated by the sequence. That is:

Theorem. *Limit Comparison Test*
Suppose $\{a_n\}$ and $\{b_n\}$ are sequences such that $a_n > 0$ and $b_n > 0$ for all n, and such that

$$lim_{n\to\infty} a_n = lim_{n\to\infty} b_n.$$

Then if $\{a_n\}$ and $\{b_n\}$ shrink at the same rate, then the series generated by these sequences, $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

These problems are meant to build intuition on this.

Problem 1. Here we sum sequences of differing growth rates, and explore the behavior of the generated series. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences such that $a_n > 0$ and $b_n > 0$ for all n, and such that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n.$$

Suppose futhermore, that $\{b_n\}$ shrinks faster than $\{a_n\}$.
[Answer two for full credit]

- If $\sum_{n=1}^{\infty} a_n$ converges, then can we determine if the series $\sum_{n=1}^{\infty}(a_n + b_n)$ converges? Why or why not?
- If $\sum_{n=1}^{\infty} a_n$ diverges, then can we determine if the series $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges? Why or why not?
- Provide sequences $\{a_n\}$ and $\{b_n\}$ so that $\sum_{n=1}^{\infty} b_n$ converges, but $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges.

0.1. **Problem 2.** Show whether the series

$$\sum_{n=1}^{\infty} \frac{\cos(n)}{2^n}$$

converges or diverges.