Big question

"How do we integrate

$$\int \sin^n x \cos^m x \, dx ?"$$

$$\int (x^2)(x^3)$$

$$= \int x^5$$

Too easy

$$\int e^x \cdot e^x$$

Some relevant techniques

- u-sub
- IBP (Integration by parts)
- "Structure of trig functions?"
  - (There are a lot of nice trig identities)
  - ($\frac{d}{dx}[\sin] = \cos$, ...)

I can't answer the question

- Is there an easier version/case that I can solve?

what is an easier version

- "maybe low powers on 1 or both"

$\cos^m x$ $\qquad$ $\sin^n x$

m: ↓ / $n =$	0	1	2	3
0	$\cancel{1}$	$\cancel{\sin x}$	$\sin^2 x$	$\sin^3 x$
1	$\cancel{\cos x}$	$\sin \cos$	$\sin^2 \cos$	
2	$\cos^2 x$		$\boxed{}$	$\boxed{}$
3	$\cos^3 x$			

$\cos^2 x$	$\sin x \quad \cos x$	$\sin^2 x$
$\cos^2 \sin x$	$\cos^3 x \quad \sin^3 x$	$\sin^2 \cos x$

$u = \sin x$

$du = \cos x$

$\int \sin x \cos x \, dx$

$= \int u \, dx$

$= \frac{1}{2} u^2 + C$

$= \frac{1}{2} \sin^2 x + C$

$\int \sin^n x \cos^m x \, dx$

$\int \sin^n x \cdot \cos x \, dx$

$\downarrow$

$\int u^n \, du$

$1 - \cos^2 x$

$= \sin^2 x$

Sub $u = \cos x$

$\int \sin x \cos x \, dx$

$= -\frac{1}{2} \cos^2 x + d$

$(d = C + \frac{1}{2})$

$\int \sin x \cos^m x \, dx$

$\downarrow$

$\int -u^m \, du$

we can now integrate

$\int \sin^n x \cos^m x \, dx$ when n or $m = 1$

$\sin^2 x + \cos^2 x = 1$

Hint: See if you can change the power on cos first.

$\sin^2 x \rightarrow 1 - \cos^2 x$

$\cos^2 x \rightarrow 1 - \sin^2 x$

$\sin^2 x \cos^2 x \rightarrow \sin^2 - \sin^4 = (\sin^2)(1 - \sin^2)$

$\sin^3 x \cos^2 x \rightarrow \sin x(\sin^2 x)\cos^2 x = \sin x(1 - \cos^2 x)\cos^2 x$

$= \sin x \cos^2 x - \sin x \cos^4 x$

$\sin^2 x \cos^3 x \rightarrow$

$\cos x \sin^2 x - \cos x \sin^4 x$

$\sin^3 x$

$\cos^3 x$

$\int \sin^3 \cos^2 x = \int (\sin x \cos^2 x) dx - \int \sin x \cos^4 x \, dx$

$\downarrow$

$u = \cos x$

$du = \sin x$

$-\int u^2 du + \int u^4 du$

$$\int \sin^3 x \cos^{\overset{m}{3}} x \, dx = \int \sin x (\sin^2 x) \cdot \cos^{\overset{m}{3}} x \, dx$$

$$= \int \sin x (1 - \cos^2 x) \cos^{\overset{m}{3}} x \, dx$$

$$= \int \sin x \cdot \cos^{\overset{m}{3}} x - \sin x \cdot \cos^{\overset{m+2}{5}} x \, dx$$

$$= \int \sin x \cos^{\overset{m}{3}} x \, dx - \int \sin x \cos^{\overset{m+2}{5}} x \, dx$$

$$\int \sin x \cos^{m} x \, dx - \int \sin x \cos^{m+2} x \, dx$$

$$\int \sin^3 x \cos^m x \, dx, \quad \int \sin^n x \cos^3 x \, dx$$

$$\sin^n \to \sin^{n-2}(1 - \cos^2)$$

$$= \sin^{n-2} - \sin^{n-2} \cos^2$$

$$\sin^{n-4} - \sin^{n-4}\cos^2 - \left(\sin^{n-4}\cos^2 - \sin^{n-4}\cos^4\right)$$

If n is odd, keep using

$$\sin^2 = (1-\cos^2)$$

applied to

$$\sin^n = \sin^{n-2}(\sin^2)$$
$$= \sin^{n-2}(1-\cos^2)$$

over and over, to reduce power on sin to 1

or look at

$$\int \sin^n x \cos^m x \, dx$$
$$= \int \sin^{n-1} x \left(\sin x \cos^m x\right) dx$$

$$\int \sin^5 x \cos^2 x \, dx$$

$$= \int \sin^4 x \, (\sin x \cos^2 x) \, dx$$

$$= \int \underline{\sin^4 x} \left(\underline{\frac{d}{dx}} \left[-\frac{1}{3} \cos^3 x \right] \right) dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \int \frac{d}{dx} \left[\sin^4 x \right] \frac{1}{3} \cos^3 x \, dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \int \frac{4}{3} \cos x \sin^3 x \cdot \cos^3 x \, dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \frac{4}{3} \left(\int \sin^3 x \cdot \cos^4 x \, dx \right)$$

$$\int \sin^3 x \cdot \cos^4 x \, dx$$

$$= \int \sin x \left(\sin^2 x\right) \cos^4 x \, dx$$

$$= \int \sin x \left(1 - \cos^2 x\right) \cos^4 x \, dx$$

$$= \int \sin x \cos^4 x \, dx - \int \sin x \cos^6 x \, dx$$

$$\Big\downarrow \; u = \cos x \qquad du = -\sin x$$

$$= -\int u^4 \, du + \int u^6 \, du$$

$$= -\frac{1}{5} u^5 + \frac{1}{7} u^7$$

$$= -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x$$