

Big question

"How do we
integrate

$$\int \sin^n x \cos^m x dx?$$

$$\int x^2)(x^3) \quad \text{To easy}$$

$$= \int x^5$$

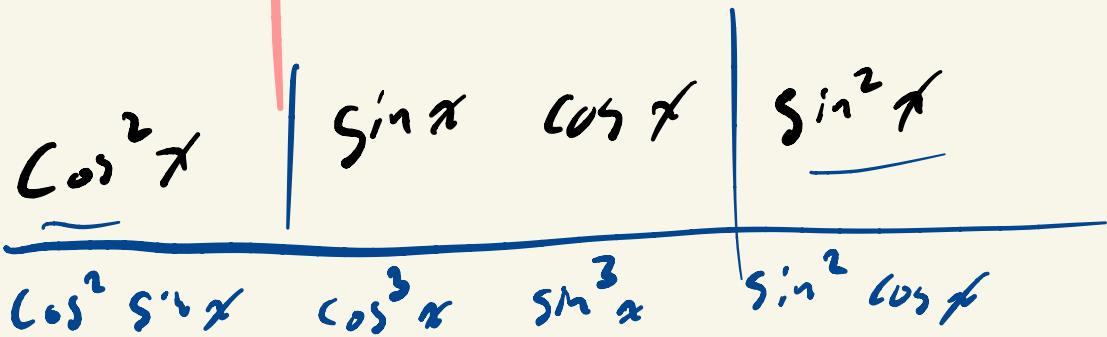
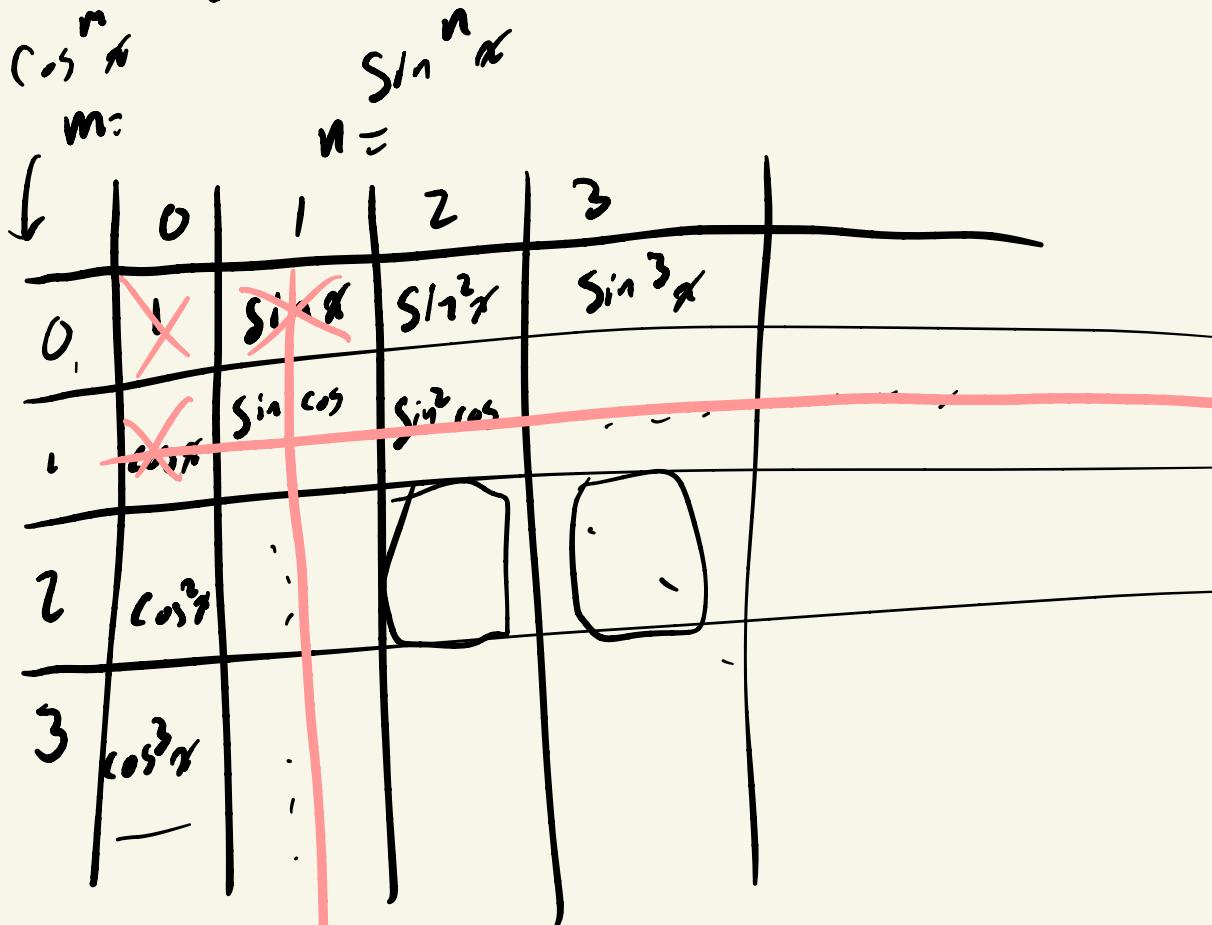
$$\int e^x \cdot e^x$$

Some relevant techniques

- U-Sub
- IBP (Integration by parts)
- "Structure of trig functions?"
 - (There are a lot of nice trig identities)
- I can't answer the question - $\left(\frac{d}{dx}[\sin] = \cos\right)$
- Is there an easier version/case that I can solve?

what is an easier way

a "maybe low powers in 10"
both



$$u = \sin x \quad \int \sin x \cos x \, dx$$

$$du = \cos x$$

$$\int u \, du = \int \sin x \cos x \, dx$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} \sin^2 x + C$$

$$\int u^n \, du$$

$$1 - \cos^2 x$$

$$= \sin^2 x$$

$$\text{Sub } u = \cos x$$

$$\int \sin x \cos x \, dx$$

$$= -\frac{1}{2} \cos^2 x + D$$

$$(D = C + \frac{1}{2})$$

$$\int \sin x \cos^m x \, dx$$

↓

$$\int -u^m \, du$$

We can now integrate

$$\int \sin^n x \cos^m x dx \quad \text{when } n \text{ or } m = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x \rightarrow 1 - \cos^2 x$$

$$\cos^2 x \rightarrow 1 - \sin^2 x$$

$$\sin^2 x \cos^2 x \rightarrow \sin^2 - \sin^4 = (\sin^2)(1 - \sin^2)$$

$$\sin^3 x \cos^2 x \rightarrow \sin x (\sin^2 x) (\cos^2 x - \sin x (1 - \cos^2 x) \cos^2 x$$
$$= \sin x \cos^2 x - \sin x \cos^4 x$$

$$\underbrace{\sin^2 x}_{\sin^3 x} \underbrace{\cos^3 x}_{\cos^3 x}$$

$$\cos x \sin^2 x - \cos x \sin^4 x$$

Hint: See if you
can change the power
on cos first.

$$\int \sin^3 \underbrace{\cos^2 x}_{\cos^3 x} = \int (\sin x \cos^2 x) dx - \int \sin x \cos^4 x$$

$$u = \cos x$$
$$du = -\sin x dx$$

$$-\int u^2 du + \int u^4 du$$

$$\begin{aligned}
 \int \sin^m x \cos^3 x dx &= \int \sin x (\sin^2 x) \cdot \cos^3 x dx \\
 &= \int \sin x (1 - \cos^2 x) \cos^3 x dx \\
 &= \int \sin x \cdot \cancel{\cos^3 x}^m - \sin x \cdot \cancel{\cos^5 x}^{m+2} dx \\
 &\quad - \int \sin x \cancel{\cos^3 x}^m dx - \int \sin x \cos^5 x dx
 \end{aligned}$$

$$\int \sin x \cos^m x dx - \int \sin x \cos^{m+2} x dx$$

$$\int \sin^3 x \cos^m x dx, \int \sin^n x \cos^3 x dx$$

$$\sin^n \rightarrow \sin^{n-2} (1 - \cos^2)$$

$$= \sin^{n-2} - \sin^{n-2} \cos^2$$

↙ ↓

$$\sin^{n-4} - \sin^{n-4} \cos^2 - \left(\sin^{n-4} \cos^2 - \sin^{n-4} \cos^4 \right)$$

If n is odd, keep using

$$\sin^2 = (1 - \cos^2)$$

applied to

$$\sin^n = \sin^{n-2} (\sin^2)$$

$$= \sin^{n-2} (1 - \cos^2)$$

Over and over, to reduce
power on \sin to 1

or look at

$$\int \sin^n x \cos^m x dx$$

$$= \int \sin^{n-1} x (\sin x \cos^m x) dx$$

$$\int \sin^5 x \cos^2 x \, dx$$

$$= \int \sin^4 x (\sin x \cos^2 x) \, dx$$

$$= \int \underline{\sin^4 x} \left(\underline{\frac{d}{dx}} \left[-\frac{1}{3} \cos^3 x \right] \right) \, dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \int \frac{d}{dx} \left[\sin^4 x \right] \cdot \frac{1}{3} \cos^3 x \, dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \int \frac{4}{3} \cos x \sin^3 x \cdot \cos^3 x \, dx$$

$$= \sin^4 x \cdot \left(-\frac{1}{3} \cos^3 x \right) + \frac{4}{3} \overbrace{\left(\int \sin^3 x \cdot \cos^4 x \, dx \right)}$$

$$\int \sin^3 x \cdot \cos^4 x \, dx$$

$$= \int \sin x (\sin^2 x) \cos^4 x \, dx$$

$$= \int \sin x (1 - \cos^2 x) \cos^4 x \, dx$$

$$= \int \sin x \cos^4 x \, dx - \int \sin x \cos^6 x \, dx$$

$$\downarrow u = \cos x \quad du = -\sin x$$

$$= - \int u^4 du + \int u^6 du$$

$$= -\frac{1}{5} u^5 + \frac{1}{7} u^7$$

$$= -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x$$