

If F is antiderivative for f ,
 G is antiderivative for g ,

$$\int_a^b Fg = [FG]_a^b - \int_a^b fG,$$

$$\int Fg = FG - \int fG$$

$$\int fg = Fg - \int F \frac{d}{dx}[g]$$

kind of like

$$\int (f)g = (f)(g) - \int f(g)$$

$$\int jh = (j)h - \int (j) \frac{d}{dx}[h]$$

If F is antiderivative for f ,
 G is antiderivative for g ,

$$\int u \, dv = uv - \int v \, du$$

$$\int F \frac{d}{dx}[G] = FG - \int G \frac{d}{dx}[F]$$

$$\int f'g \, dx = fg - \int fg' \, dx$$

$\int$ is a family of functions

e.g. $\int + C$

- $\int x \cdot \arctan(x) \, dx$

- $\int \ln(x) \, dx$
- $\int (x^3 + x^5) \ln(x) \, dx$

$$\int x \arctan(x)$$

maybe? $((\sqrt{x}) (\sqrt{x} \arctan(x)))$

$$(x) (\arctan(x))$$

$$\int \frac{\frac{1}{2}x^2 + C}{x^2 + 1}$$

$$C = \frac{1}{2}$$

↓

$$\frac{1}{2} \left(\frac{x^2 + 1}{x^2 + 1} \right)$$

$\frac{d}{dx}$	$\frac{d}{dx}$	0
$\frac{d}{dx}$		1
x		x
$\int$		$\frac{1}{2}x^2 + C$
$\int \int$		

$\frac{d}{dx}$	$\frac{d}{dx}$	
$\frac{d}{dx}$		$\frac{1}{1+x^2}$
$\arctan$		$\arctan(x)$
$\int$		
$\int \int$		

$$\left(\int f' g \, dx = f g - \int f g' \, dx \right)$$

$$\int x \arctan(x)$$

$$= \int \underbrace{\frac{d}{dx} \left[\frac{1}{2} x^2 + \frac{1}{2} \right]} \arctan(x)$$

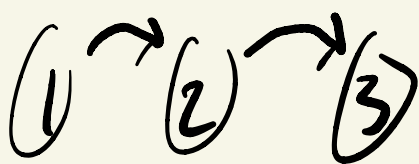
$$= \left(\frac{1}{2} x^2 + \frac{1}{2} \right) \arctan(x) - \int \frac{1}{2} \left(\frac{x^2 + 1}{x^2 + 1} \right)$$

$$= \left(\frac{1}{2} x^2 + \frac{1}{2} \right) \arctan(x) - \frac{1}{2} x.$$

② Polynomial $\rightarrow$ (algebraic)

① trig, exponential

③ log inverse trig
 $\uparrow \quad \quad \uparrow$



move $\frac{d}{dx}$ to the right

$$\int f(x) g(x) dx$$

$$\left(\frac{x^2+1}{x^3+2} \right)$$

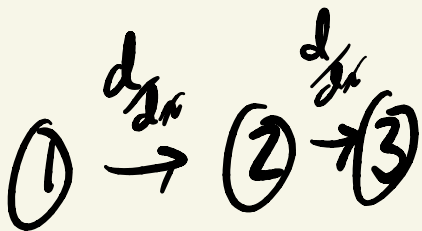
$$\left(\frac{\sqrt{x^2+1}}{x^3+2} \right)$$

• Polynomial or rational or algebraic (2)

• inverse trig or log (3)

• trig & e^x (1)

$$\frac{d}{dx} [\ln x] = \frac{1}{x}$$



"move $\frac{d}{dx}$ to the right w/ IBP"

LIATE

$\int$
 $\frac{d}{dx}$
 $\ln$
 g
 x

"For 'Int' move.
 $\frac{d}{dx}$ to the Left"

$$\int \frac{d}{dx}[f(x)] g(x) dx$$

$$= f(x)g(x) - \int f(x) \frac{d}{dx}[g(x)] dx$$

$$\int \ln(x) dx$$

$$\int (x^3 + x^5) \ln(x) dx$$

$$\frac{d}{dx} \left[\frac{x^4}{4} + \frac{x^6}{6} + C \right] = x^3 + x^5$$

$$\int \frac{d}{dx} \left[\frac{x^4}{4} + \frac{x^6}{6} + C \right] \cdot \ln(x) dx$$

$$\frac{d}{dx} [\ln(x)]$$



$$= \left(\frac{x^4}{4} + \frac{x^6}{6} + C \right) \cdot \ln(x) - \int \left(\frac{x^4}{4} + \frac{x^6}{6} \right) \cdot \frac{1}{x} dx$$

$$= \left(\frac{x^4}{4} + \frac{x^6}{6} + C \right) \cdot \ln(x) - \int \left(\frac{x^3}{4} + \frac{x^5}{6} + \frac{C}{x} \right) dx$$

$$\stackrel{C=0}{=} \left(\frac{x^4}{4} + \frac{x^6}{6} \right) \ln x - \int \left(\frac{x^3}{4} + \frac{x^5}{6} \right) dx$$

$$\begin{aligned}
 \int 1 \cdot \ln(x) dx &= \int \underbrace{\frac{d}{dx}[x]}_{1} \cdot \ln(x) dx \\
 &= x \ln(x) - \int x \frac{d}{dx}[\ln(x)] dx \\
 &= x \ln(x) - \int 1 dx \\
 &= x \ln(x) - x + C
 \end{aligned}$$

$$\frac{d}{dx} [x \ln x - x]$$

$$\ln x + 1 - 1 = \ln x$$