THINGS YOU NEED TO KNOW: BETWEEN MIDTERM 1 AND MIDTERM 2

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This is a list of important information covering topics that lie between midterms 1 and 2. This is not a comprehensive list of things that may be on the exam, but rather a list of things which you certainly should know.
This list will be continually updated as we learn more.

Conceptual Skills

- You should be able to relate graphical properties of a function with graphical properties of the function's derivatives.

Derivatives of Special Functions

- **Power Rule:**
  For any real number n with $n \neq 0$,
  $$\frac{d}{dx}\Big(x^n\Big) = nx^{n-1}.$$
  For $n = 0$,
  $$\frac{d}{dx}\Big(x^0\Big) = 0.$$

- **Derivatives of exponential functions:**
  If a is some real number with $a > 0$, then
  $$\frac{d}{dx}\Big(a^x\Big) = (\ln a) \cdot a^x.$$
  Notice that this implies
  $$\frac{d}{dx}\Big(e^x\Big) = e^x.$$

- **Derivatives of trigonometric functions:**
The following equalities hold:
$$\frac{d}{dx}\Big(\sin(x)\Big) = \cos(x) \quad \text{and} \quad \frac{d}{dx}\Big(\text{cox}(x)\Big) = -\sin(x)\Big).$$
From these we obtain the following equalities:
  * $\frac{d}{dx}\Big(\tan(x)\Big) = \sec^2(x)$
  * $\frac{d}{dx}\Big(\cot(x)\Big) = -\csc^2(x)$
  * $\frac{d}{dx}\Big(\sec(x)\Big) = \tan(x)\sec(x)$
  * $\frac{d}{dx}\Big(\csc(x)\Big) = -\cot(x)\csc(x)$

Rules for Constructing New Derivatives from Old

- **Sum Rule:**
If f and g are functions which are differentiable at some point a, and h is a function defined by $h(x) = f(x) + g(x)$, then
$$h'(x) = f'(x) + g'(x).$$

This is probably the most straightforward differentiation rule to use, it lets us differentiate the sum of functions by individually differentiating the summands.

- **Constant Multiple Rule:** If f is a function which is differentiable at some point a, and c is any real number, then
$$\frac{d}{dx}\Big(cf(x)\Big) = c\Big(\frac{d}{dx}f(x)\Big).$$

This together with the power rule and sum rule allows us to easily differentiate polynomials.

- **Product Rule:** If f and g are functions which are both differentiable at some point a, and h is the function defined by $h(x) = f(x) \cdot g(x)$, then
$$h'(a) = f(a) \cdot g'(a) + f'(a) \cdot g(a).$$

- **Quotient Rule:** If f and g are functions which are both differentiable at some point a with $g(a) \neq 0$, and h is the function defined by $h(x) = \frac{f(x)}{g(x)}$, then
$$h'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{(g(a))^2}.$$

- **Chain Rule:** If f and g are functions such that g is differentiable at a and f is differentiable at $g(a)$ then if $h(x) = f(g(x))$ we have
$$h'(a) = f'(g(a)) \cdot g'(a).$$