CALC 1 CHALLENGE PROBLEM BANK

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This is a list of challenging or interesting problems which can be solved with knowledge from Calc 1. Problems are grouped by how far along into the course you see the information most relavent to the problem. Some of these problems may be able to be solved with less information.

For the MATH 1300 Section 2 Fall 2019 class, if you submit up to five of these problems to Ian with adequate solutions you can may recieve up to 10 points of extra credit towards friday quizzes by replacing your lowest grade as follows.

- 1 problem: Lowest grade below 2/10 will be replaced with a grade of 2/10.
- 2 problems: Lowest grade below 4/10 will be replaced with a grade of 4/10.
- 3 problems: Lowest grade below 6/10 will be replaced with a grade of 6/10.
- 4 problems: Lowest grade below 8/10 will be replaced with a grade of 8/10.
- 5 problems: Lowest grade below 10/10 will be replaced with a grade of 10/10.

If you are submitting problems, you may use the resources available to you, and it is likely that some solutions can be found online. I do reserve the right to have you present a problem to the class if time permits and I may ask you questions in person related to the solution.

Material Up To Exam 1.

- Following is a useful inequality.
  For any real numbers $\alpha \geq 1$, and any n numbers $a_1, a_2, \ldots, a_n \geq 0$, the following inequality holds
  $$a_1^\alpha + a_2^\alpha + \cdots + a_n^\alpha \leq (a_1 + a_2 + \cdots + a_n)^\alpha.$$
  Show that this inequality holds.
  Hints:
  - Consider the functions involved here
  - What is true about secant lines of concave up functions?
  - Try to first solve this for $n = 2$, that is show that under the hypothesis we have
  $$a_1^\alpha + a_2^\alpha \leq (a_1 + a_2)^\alpha.$$
- Can we establish a Squeeze Theorem for derivatives? Answer the following questions
  - Given a function f such that $-x^2 \leq f(x) \leq x^2$ for all real numbers x, can we determine the derivative of f at 0?
  - How can we generalize this into a general statement? Show that your statement is true.
- Construct a function f such that f is differentiable for all real numbers, but such that f' is not continuous for all real numbers. There is a standard example which uses trig functions, try to construct an example using piecewise polynomial functions.
  Hint: What kinds of discontinuities could f' have?

- For this question we will say that a function f "grows faster" than a function g if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty.$$

Show that $f(x) = e^x$ grows faster than $g(x)$ when $g(x)$ is any polynomial.
Hints:
  - Can you first show that e^x grows faster than x? How about x^2
  - For any polynomial g, is there some natural number n such that x^n grows faster than g?