HOMEWORK 3

1. Problems

(1) Describe the following quotient spaces:
   (a) the cylinder with its boundary circles identified to a point
   (b) the torus with the subset consisting of one meridianal and one longitudinal circle identified to a point
   (c) the 2-sphere with the equator identified to a point
   (d) The plane $\mathbb{R}^2$ with each of the circles center the origin and of integer radius identified to a point.

(2) Which space do we obtain if we take a Möbius strip and identify its boundary to a point?

(3) Let X denote the unions of the circles

$$(x - 1/n)^2 + y^2 = 1/n^2,\ n = 1, 2, 3...,$$

with the subspace topology induced from the standard topology in $\mathbb{R}^2$. Let Y be the quoitient space obtained from the real line by $(\mathbb{R}, ST)$ by identifying all of the integers to a point, with quotient topology. Show that X and Y are not homeomorphic. (You are allowed some hand-waving in this problem since we have not covered yet all the necessary tools for a totally rigorous proof)

(4) Give an example of quotient space in where the quotient map is neither open nor closed.