HOMEWORK 2

1. Problems

(1) Let $(X, \mathcal{T})$, $(Y, \mathcal{W})$ be two topological spaces, and

$$f: \ X \to Y$$

be a function. Prove that the following are equivalent
   (a) f is continuous
   (b) $f(\overline{A}) \subseteq \overline{f(A)}$, $\forall A \subseteq X$.
   (c) $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, $\forall B \subseteq Y$.

(2) Let $(X, \mathcal{T})$, $(Y, \mathcal{W})$ be two topological spaces, and

$$f: \ X \to Y$$

be a function. Prove that the following are equivalent
   (a) f is continuous
   (b) $f^{-1}(I(B)) \subseteq I\Big(f^{-1}(B)\Big)$, $\forall B \subseteq Y$. Herere $I(C)$ means the interior of the set C.