SOLUTION to Problem 12 of the final:

Direct computation is messy and therefore one either uses the Divergence Theorem or Stokes's Theorem twice.

- **Solution with Divergence Thm:** First, recall that $\mathsf{div}\vec{G} = \mathsf{div}\,\mathsf{curl}\vec{F} = 0$. Therefore the outward flux through the given surface equals the inward flux through the 'lid' which is a disk at $y = 1$. For the inward flux, we should consider the inward unit normal $\langle 0, -1, 0\rangle$, and since $\mathsf{curl}\vec{F}$ is such that its second component is 3, the inward flux through the disk equals -3 times the area of the disk, that is -3π.
- **Solution with Stokes:** The closed curve $\{(x, y, z) \mid y = 1, x^2 + z^2 = 1\}$ is the common boundary of both $\mathcal{S}$ and the disk. Therefore, applying Stokes *twice*, it follows that the outward flux through the given surface equals the inward flux through the disk, and we continue as before. $\square$

Note: Using Stokes, one may choose to simply compute the line integral of $\vec{F}$ on the closed curve as well, but that's more difficult.