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Time, the Grand Illusion

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The reconcilability of gravitational with electromagnetic clocks suggests that a rigorous analysis of time will provide understanding of the unity of gravity and electromagnetism. Time is found to be fundamentally a property of elementary particles, only derivatively a property of clocks. A declaration is made: that the flow of an elementary particle's time is *the change of its radius, that time is therefore illusory. The de Sitter expanding universe is derived from this principle by treating elementary particles as spheres in Euclidean space. The hyperspheres of de Sitter space call up a five-dimensional metric manifold whose geometry models gravity, electromagnetism, and other phenomena tied to the structure of matter; neutrinos are provided for. Distance in this manifold is related to a secondary time, not correlated to primary time, but just as illusory. A particle's inertial rest mass is the relative rate of its two proper times; mass and charge are jointly, not individually, conserved.*

Grandfather's pendulum clock and grandson's tuning fork wrist watch tick away the seconds in near perfect unison. This remarkable but rarely remarked coincidence[1] constitutes transparent and incontrovertible evidence of the inseparability of the gravitational and the electromagnetic fields, the respective principal governors of those timetellers. Moreover, it strongly suggests that an understanding of this unity will issue from a thoroughgoing analysis of the nature of time. Finally, it hints that the same analysis will elucidate the

[1] The only published reference to the significance of this coincidence that I have found is quite recent.[1]

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concept of inertial mass; for both pendulum and tuning fork possess it, but the tuning fork's second is conditioned by how much, whereas the pendulum's is not.

Is time exclusively a property of clocks? Certainly not of any individual clock if by "clock" is meant a physical system whose states periodically repeat themselves, for even to speak of this repetition requires reference to a time kept elsewhere. Nor will any finite set of clocks suffice, for then one would have a vicious circle of clocks, each waiting for the next to start ticking. It is possible to avoid this circle by imagining as infinite and without end the descending hierarchy of clocks that begins with galaxy, solar system, earth–moon system, rotating earth, swinging pendulum, ringing crystal, vibrating molecule, atom,.... Timekeepers that are not clocks would not then be required. To do this, however, is in part to deny the existence of truly elementary particles and thus to reject the general relativistic or geometrodynamical model of the world, in which such particles appear unbidden as the topological holes in space. If this model is valid—and I proceed on the supposition that it is—then the hierarchy must end with elementary particles that perhaps themselves serve as clocks (by rotating, for example), but, beyond that, produce in some mysterious way a fundamental time to which clocks may be referred. One must seek the source of that time.

The topological holes that geometrodynamically represent isolated, nonrotating elementary particles possess as a common feature smallest spherical cross sections.[2–5] For a specific particle the smallest cross-sectional radius may vary with time, as in, for example, the Schwarzschild and the Reissner–Nordström wormholes. Seizing the rod by the other end, one can say that it is, really, that time varies with the radius. I go further. I bend the rod into a circle (nonvicious) and offer a radical and far-reaching declaration: The variation of an elementary particle's time *is* the variation of its radius;

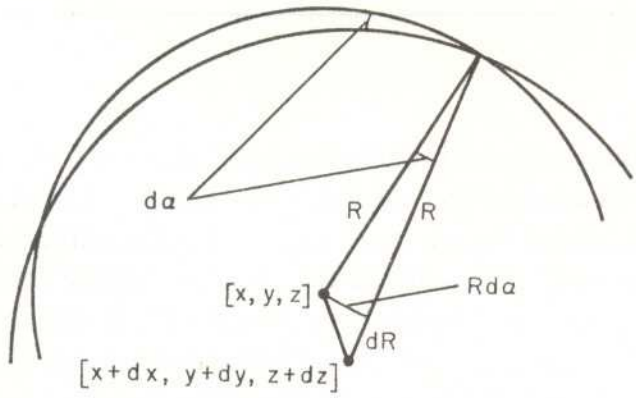

Fig. 1. Two neighboring spheres in Euclidean three-space, intersecting in the angle $d\alpha$—shown in cross section through their centers.

time is size and size is time; old is big and young is small, or vice versa; time is but a grand illusion!

Pure fancy, one *might* say. So permit me to tender persuasions. In the space of Euclid let there be two neighboring spheres, of centers $[x, y, z]$ and $[x + dx, y + dy, z + dz]$, and of radii R and $R + dR$, intersecting as in Fig. 1. The infinitesimal angle $d\alpha$ in which they intersect is seen by an elementary consideration to satisfy

$$d\alpha^2 = (1/R^2)(dx^2 + dy^2 + dz^2 - dR^2) \tag{1}$$

This expression is a conformal invariant, dependent only upon ratios of distances, thus independent of scale. It has meaning even when the spheres do not intersect, in which case $d\alpha$ is imaginary. It constitutes a metric for the four-dimensional manifold of Euclidean spheres, coordinatized by x, y, z, and R. Let $t = -\ln R$. Then $R = e^{-t}$, and

$$d\alpha^2 = e^{2t}(dx^2 + dy^2 + dz^2) - dt^2 \tag{2}$$

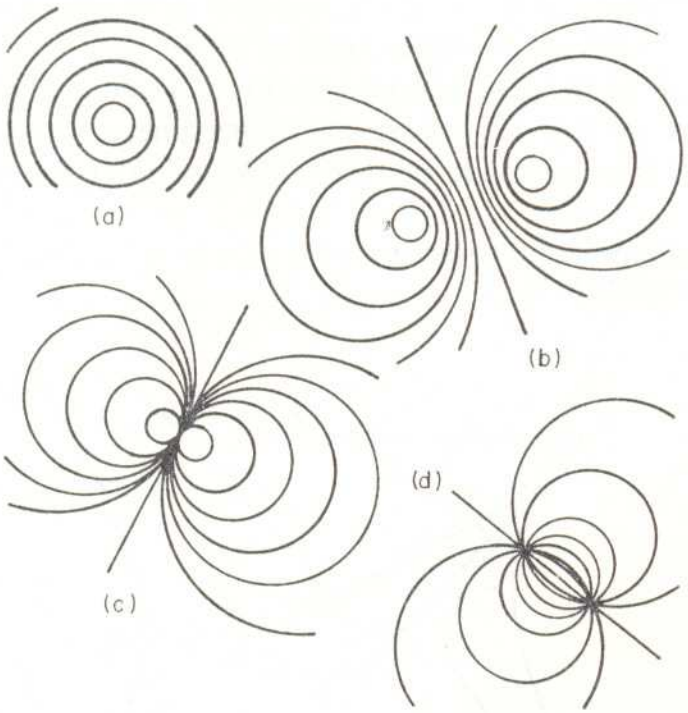

Fig. 2. One-parameter families of Euclidean-space spheres (shown in centered cross section) representing typical geodesics of the angle metric, and corresponding to geodesics of de Sitter space; (a) and (b) are pretangential (timelike, subluminal), (c) is tangential (lightlike, luminal), (d) is posttangential (spacelike, superluminal). For completeness the Euclidean-space planes are included as spheres of infinite radius.

But we recognize this as the line element of the de Sitter expanding universe in the Lemaître frame.[6] The cosmic time t to which the expansion is referred is nothing other than the time derived from the radii of the spherical elementary particles in accordance with the new declaration. Does the universe expand? Or is it that the particles shrink? It is a relative matter. What counts is that the radii of the spheres that represent a free particle's existence in Euclidean space, corresponding to the events on its geodesic world-line in the de Sitter picture, progress from large to small, as illustrated in Fig. 2. According to the declaration, neither more nor less than this progression is required for time to flow.

What of proper time τ, defined by

$$d\tau^2 = dt^2 - e^{2t}(dx^2 + dy^2 + dz^2) \tag{3}$$

For a particle of fixed center $[x, y, z]$ (Fig. 2a) $\tau = t$. For each other subluminal, or pretangential particle (Fig. 2b) there is a conformal mapping of space which "brings it to rest," that is, converts its world-line to a family of concentric spheres, whose radii R' relate directly to τ via $\tau = -\ln R'$.[2] Thus the "time" that the declaration refers to is none other than the particle's "proper time," so named by Minkowski in 1908.[7]

In the space of spheres, metrized by Eq. (1)—equivalently, in de Sitter space, metrized by Eq. (2)—there are hyperspheres (hyper-hyperboloids, actually). What is to be made of *their* angles of intersection? To treat the

[2] This mapping corresponds to an isometry of the de Sitter universe, a "transformation to a rest frame of the particle."

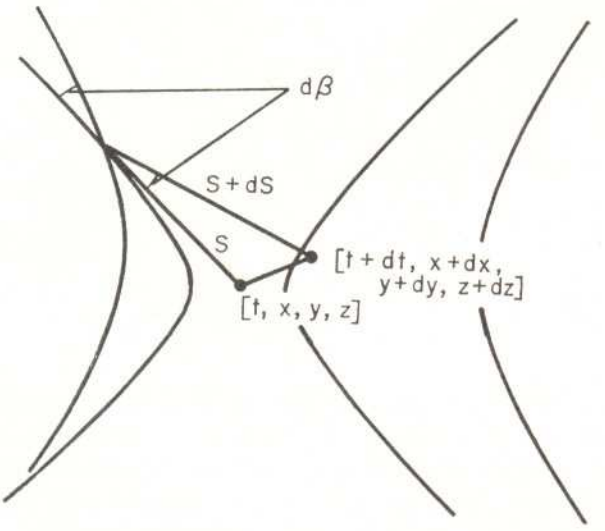

Fig. 3. Two neighboring, one-sheeted hyperspheres of Minkowski space, intersecting in the angle $d\beta$—shown in centered cross section.

problem in a simplified, yet illuminating context, consider in Minkowski space (or in a small, essentially Minkowskian region of de Sitter space) two neighboring hyperspheres of the one-sheet variety, with centers $[t, x, y, z]$ and $[t + dt, x + dx, y + dy, z + dz]$ and radii S and $S + dS$, intersecting in the angle $d\beta$ as in Fig. 3. This angle $d\beta$ is the basic differential invariant of the conformal group of Minkowski space, which is the maximal invariance group of Maxwell's equations.[8,9] One has that

$$d\beta^2 = (1/S^2)(dS^2 + dt^2 - dx^2 - dy^2 - dz^2) \tag{4}$$

and hence that

$$d\tau_2{}^2 = dt_2{}^2 + e^{2t_2}(dt_1{}^2 - dx^2 - dy^2 - dz^2) \equiv e^{2t_2}d\tau_1{}^2 + dt_2{}^2 \tag{5}$$

where $t_1 = t$, $t_2 = -\ln S$, and $\tau_2 = \beta$. The transition from Eq. (4) to Eq. (5) is just one small step across a mathematical ditch. But the clear analogy with what has gone before urges one on toward a giant leap across a profound psychological chasm, from whose other side the world will seem strange and surreal. I assert, in extension of the declaration, that, for each elementary particle, τ_2 is a higher order, or secondary, proper time, different in rank, but not different in kind from the particle's familiar, primary proper time τ_1, equally deserving of the name "proper time," and equally illusory. To accept this is to leap the abyss. As one debates making the jump, a report from a scout on the other side may be welcomed and weighed.

The most general five-dimensional metric with the signature of the line element (5) can be expressed by

$$\begin{aligned} \bar{G} &= G + e^{2\psi}A \otimes A \\ &\equiv g_{\mu\nu}\,dx^\nu \otimes dx^\mu + e^{2\psi}(A_\nu\,dx^\nu + dx^4) \otimes (A_\mu\,dx^\mu + dx^4) \end{aligned} \tag{6}$$

where $\mu, \nu = 0, 1, 2, 3$, and G has diagonal signature $+---$. To incorporate into $\bar{G}$ the conformal aspect of (5) requires the constraint $g_{\mu\nu}(x^\kappa, x^4) = \{\exp[2C(x^\kappa, x^4)]\}\, h_{\mu\nu}(x^\kappa)$, equivalently expressed by a Lie derivative as

$$\mathscr{L}_{\partial/\partial x^4}G = 2\phi G \tag{7}$$

where $\phi = \partial C/\partial x^4$. The histories of (test) elementary particles are represented, presumptively, by the geodesic paths of the metric $\bar{G}$ in its carrying manifold $\mathscr{M}$, whose points correspond in principle to the hyperspheres of the space of spheres of a Riemannian three-space, such as the space of Euclid. The velocity components of such a path p satisfy the geodesic equations

$$(\dot{p}^\mu)^{\cdot} + \dot{p}^\kappa \left\{ {\mu \atop \kappa\lambda} \right\} \dot{p}^\lambda = e^\psi \dot{p}^4 \dot{p}^\kappa F_\kappa{}^\mu - 2e^{-\psi}\phi \dot{p}^4 \dot{p}^\mu + \dot{p}^4 \dot{p}^4 B^\mu \tag{8}$$

and

$$(\dot{p}^4)^{\cdot} + \dot{p}^4 \dot{p}^\kappa B_\kappa = e^{-\psi}\phi \dot{p}_\kappa \dot{p}^\kappa \tag{9}$$

Here $\dot{p} = \dot{p}^\mu e_\mu + \dot{p}^4 e_4$, where $\{e_\mu, e_4\}$ is the frame field dual to $\{dx^\mu, e^\psi A\}$. Also,

$$\{_\kappa{}^\mu{}_\lambda\} = \tfrac{1}{2}[(\partial g_{\nu\lambda}/\partial x^\kappa) + (\partial g_{\kappa\nu}/\partial x^\lambda) - (\partial g_{\kappa\lambda}/\partial x^\nu)]\, g^{\nu\mu} \tag{10}$$

$$F_\kappa{}^\mu = F_{\kappa\lambda} g^{\lambda\mu}, \quad \text{where} \quad F_{\kappa\lambda} = (\partial A_\kappa/\partial x^\lambda) - (\partial A_\lambda/\partial x^\kappa) \tag{11}$$

$$B^\mu = g^{\mu\nu} B_\nu, \quad \text{where} \quad B_\nu = (\partial\psi/\partial x^\nu) - (\partial A_\nu/\partial x^4) \tag{12}$$

and

$$\dot{p}_\kappa = g_{\kappa\lambda}\dot{p}^\lambda \tag{13}$$

When the path parameter is secondary proper time, then $\cdot \equiv d/d\tau_2$.

Let

$$m = (\dot{p}_\mu \dot{p}^\mu)^{1/2} \tag{14}$$

and

$$q = \dot{p}^4 \tag{15}$$

Then $m = \dot{\tau}_1 = d\tau_1/d\tau_2$, and if $u^\mu = dp^\mu/d\tau_1 \equiv \dot{p}^\mu/\dot{\tau}_1$, then Eq. (8) is equivalent, wherever $m \neq 0$, to

$$\frac{d}{d\tau_1}(mu^\mu) + (mu^\kappa)\{_\kappa{}^\mu{}_\lambda\}\, u^\lambda = e^\psi q u^\kappa F_\kappa{}^\mu - 2e^{-\psi}\phi q u^\mu + \frac{q^2}{m} B^\mu \tag{16}$$

Now I identify m as the inertial "rest" mass and q as the electric charge of the elementary particle. Equation (16) states that the G covariant derivative, with respect to *primary* proper time, of the four-momentum mu^μ is coupled to three apparent forces: the Lorentz force $qu^\kappa F_\kappa{}^\mu$ on the charge q due to the electromagnetic field $F_{\kappa\lambda}$, the velocity-proportional damping force qu^μ on the charge q, and the force $(q^2/m)\, B^\mu$ on q due in part to the vector potential A directly, without mediation of F. The gravitational force is contained, as usual, in the terms $(mu^\kappa)\{_\kappa{}^\mu{}_\lambda\}\, u^\lambda$. Besides this equation, there is the charge evolution equation

$$(dq/d\tau_1) + q(u^\kappa B_\kappa) = m\phi e^{-\psi} \tag{17}$$

equivalent to Eq. (9).

Through these equations the analysis of time provides us with an understanding of the unity of gravity and electromagnetism, a definition of inertial mass, a definition of electric charge, and unexpected forces capable in principle of shedding new light on radiation damping and on the Bohm–Aharonov effect.[10,11] It shows inertial rest mass and electric charge to be generally not constants, but kinematical variables satisfying the enlarged conservation law, consequent upon Eqs. (14) and (15), that

$$m^2 + q^2 = |\,\dot{p}\,|^2 = \text{const} \tag{18}$$

It provides a place for a massless, chargeless neutrino, for $m \equiv 0$ and $q \equiv 0$ are easily seen to be consistent with Eqs. (8), (9), and (18). On the other hand, it insists that photons are charged (how, otherwise, could the electromagnetic field influence them?), that in fact they are electrons incognita, for the geodesics with $m^2 + q^2 > 0$ can have $m = 0$, $q \neq 0$ at some points and $m \neq 0$, $q \neq 0$ at others. Indeed, electrons can travel in the guise of charged tachyons ($m^2 < 0$, $q^2 > 0$), for light speed is not a barrier in these equations[3]; the photon barrier $m^2 = 0$ has vanished, only to be replaced by the neutrino barrier $m^2 + q^2 = 0$. The picture is strange, but is internally consistent. It may be compared word for word with the orthodox picture, but cautiously, for the words now have meanings more complex and more pregnant than before.

The scout has ranged ahead and returns with news of even stranger vistas, imperfectly perceived, but reasonably clear in their broader outlines. A hole was seen gliding through space, passing away its secondary seconds. It was expanding, and its charge growing. Overcome by tensions of space around an atomic nucleus it was approaching, it began to contract. At this moment its inertial rest mass vanished, and its primary proper time, which had been decreasing, began instead to increase. The hole continued toward the nucleus, but came to a halt near it as its charge decayed to zero and its mass grew to one. As he turned away the scout thought he saw, from the corner of his eye, the hole split into two, of which one slipped away and the other came to rest closer to the nucleus. A nearby physicist recorded in his logbook that an electron–positron pair had been created, that the positron had been pushed away from the nucleus, and that the electron had been captured and had subsequently emitted a photon and dropped to a lower position.

The two views are not at variance. To make primary and secondary time advance together, it is necessary to reverse the direction of secondary time in the particle's expansionary phase. This reverses the sign of q and thereby shows the Lorentz force on the charge to be repulsive in this phase if attractive in the other. The electron is the hole entering captivity as primary and secondary time advance; the positron is the hole escaping as primary time advances and secondary time retreats; at the instant when the pair first appears in primary time the hole is a photon. The electric charge of the electron–photon–positron is a measure of its responses to the varying spatial tensions it encounters; the nucleus is to be thought of not as electrically

[3] One may not, however, expect the electromagnetic field alone to accelerate an electron to a photon or a tachyon, if light speed *is* a barrier to the field; the wind wins its race with the wave it pushes, and the wave with the surfer it carries, except the surfer catch the wind in a sail.

charged, but only as surrounded by charge-inducing tensions in space.[4]

A vast speculation now emerges. Organic life is characterized by the establishment and preservation of molecular memories, most notably in the genes of species, but in the individual organisms as well. To build these molecules requires that the elementary particles contract, hence that their surroundings should appear to expand. Organic life therefore ages (grows more memory-laden) only in concert with the *advance* of primary time; it is this that determines our subjective sense of "the arrow of time," and not, as is commonly thought, the increase of thermodynamic entropy.

Other strange sights have been glimpsed, but will not be related here. Now I leave you to your decision, but with an "unscientific" admonishment: If you are wavering on the brink, persuaded toward, but not convinced of the illusoriness of primary proper time and the existence of an equally illusory secondary proper time, then try to think of a reason *not* to accept them that is not grounded in an instinctive recoil from the psychological abyss. If, on the other hand, you are already planning to build a secondary clock, and wish to look at that task from every angle, then pause to reflect upon this: the sphere-angle-hypersphere-angle construction can be extended—time after time after time...

ACKNOWLEDGMENTS

Some of the ideas and images that have here been extracted from my hypothesis on the nature of time are, though conceived anew by myself, of earlier origin. Various anticipations can be seen in the writings of H. Weyl, Th. Kaluza, O. Klein, E. C. G. Stueckelberg, R. P. Feynman, H. C. Corben, R. L. Ingraham, and H. Reichenbach. Also, the idea that the flow of time is an illusion of some sort goes back to Plato (427 ?–347 B.C.) and to Parmenides before him.

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[4] The scout's second report and the resolution of its apparent conflict with the customary view is based upon a study of the geodesics of the metric (6) with A taken to be a Coulomb potential, namely $A = x^4(a/\rho)\,dt + dx^4$, with $\psi = 0$, and with $C = x^4\phi(\rho)$.

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C o r r i g e n d a t o

TIME, THE GRAND ILLUSION

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Equations (10), (11), and (12) should read as follows:

$$\left\{ {}_{\kappa}{}^{\mu}{}_{\lambda} \right\} = \frac{1}{2} \left[e_{\kappa} g_{\nu\lambda} + e_{\lambda} g_{\kappa\nu} - e_{\nu} g_{\kappa\lambda} \right] g^{\nu\mu} \qquad (10)$$

$$F_{\kappa}{}^{\mu} = F_{\kappa\lambda} \, g^{\lambda\mu} \quad , \quad \text{where} \quad F_{\kappa\lambda} = e_{\lambda} A_{\kappa} - e_{\kappa} A_{\lambda} \qquad (11)$$

$$B^{\mu} = g^{\mu\nu} \, B_{\nu} \quad , \quad \text{where} \quad B_{\nu} = e_{\nu} \psi - e^{\psi} e_{4} A_{\nu} \qquad (12)$$

These corrections do not affect the conclusions of the article.