Leptons might not generate gravity

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Abstract

A simple thought experiment suggests that, contrary to assertions in an earlier Letter, constancy across materials of the ratio of active to passive gravitational mass does not rule out that electrons (and other leptons) could have active gravitational mass zero, thus might not generate gravity. If they do not, then widely held assumptions about the gravitational effects of various forms of energy cannot be sustained.

Key words: Active gravitational mass; Passive–inertial mass; Leptons; Energy conditions; Drainhole; Traversable wormhole

A September 2001 Letter [1] argues that improvements in the sensitivities of certain experiments, that of Kreuzer [2] in particular, could settle the question whether leptons generate gravity. The argument has two parts. The first says that such improvements could establish more firmly that the ratio of active gravitational mass to passive gravitational mass (thus to inertial mass) is the same for all material bodies. (The Letter refers to the uniformity of this ratio as 'equality' of the masses, and takes the ratio to be 1, as can be arranged by a suitable choice of unit for the active mass.) The second part of the argument, which is not made explicit, says that constancy of this ratio across materials would require that not only the baryons but also the leptons in atoms generate gravity. The first part is unexceptionable, but the second involves an unjustified hidden assumption, which can be exposed in the following way.

As Eq. (2) of the Letter is presented, it states that if the electrons in a material body do not generate gravity, but everything else in it does, then the active mass M_a of the body is its passive–inertial mass $M_{p,i}$ reduced by the sum $M^e_{p,i}$ of the passive–inertial masses of its electrons: $M_a = M_{p,i} - M^e_{p,i}$. Because the experiments in question can at best confirm equality only between the active mass and the passive–inertial mass of the *whole body*, this equation is not justified — but it is not the unjustified assumption referred to above. Let us

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replace Eq. (2) by an equation that would be justified by such confirmation of equality, namely that $M_{p,i} = M_a = M_a^e + M_a^r$, where M_a^e is the sum of the active masses of the electrons and M_a^r is the sum of the active masses of the remaining constituents of the body (which masses are presumed to be additive quantities).[2] Now the unstated part of the argument of the Letter, if made explicit, would be that two homogeneous, geometrically congruent, electrically neutral bodies B and B′, made of different materials but having by design $M_{p,i} = M'_{p,i}$, and by observation $M_a = M'_a$, thus $M_a^e + M_a^r = M_a^{e\prime} + M_a^{r\prime}$, would have nucleons in equal number, but different numbers of protons, thus different numbers of electrons, thereby ruling out the possibility that $M_a^e = M_a^{e\prime} = 0$, *unless* the atomic and molecular binding energies in B and B′ generate gravity in just the amounts required to balance the equation $M_a^r = M_a^{r\prime}$. This is where the unjustified assumption is hiding.

Consider, for clarity, the following thought experiment: A single, isolated hydrogen atom, comprising one proton and one electron, is approached by an antineutrino. In a miraculous occurrence of reverse β-decay the antineutrino grabs the electron and disappears with it into the proton, thereby converting the hydrogen atom into a neutron. If the electron and the antineutrino (leptons both) have active gravitational mass zero, would the neutron's gravitational field differ in any way from that of the hydrogen atom? If so, then according to conventional theory the difference must be attributed to changes in the system's energy and passive–inertial mass. But if electrons, which have nonzero passive–inertial mass, don't gravitate, then the same may be true of other manifestations of passive–inertial mass and, in light of $E = m_{p,i}c^2$, of some forms of energy as well, in particular of those that changed in the transition from hydrogen atom to neutron. It is thus perfectly consistent with the proposition that leptons do not generate gravity to *not* assume that the gravitational field will differ for the hydrogen atom and the neutron. That it will differ is the hidden assumption in its barest form.

Let us extend this analysis to the Kreuzer experiment. That experiment compared the gravitational attraction exerted on test objects by each of two homogeneous, geometrically congruent, electrically neutral bodies A and B, differently constituted but weighing the same, thus having the same passive–inertial mass $M_{p,i}$. The precision of the measurements allowed the inference that the ratios of active to passive–inertial mass for the two bodies differed by less than 5×10^{-5}. Again for clarity, consider an idealized version of the experiment in which body A is made of a single isotope of one element, each of whose atoms has p_{A} protons, the same number of shell electrons, and n_{A} neutrons, and

[2] The 'additivity' of active masses mentioned here, though precise in a linear theory of gravity such as Newton's, must be understood in a nonlinear theory such as Einstein's as referring to a nonlinear superposition of gravitational effects that, at points an experimentally reasonable distance from the body, can be treated as linear. This is an assumption not treated in the previous Letter and not to be treated in the present Letter; it too is not the unjustified hidden assumption in question.

body B is made of a single isotope of another element, each atom of which has p_{B} protons and shell electrons, and n_{B} neutrons, with $p_{\mathrm{A}} + n_{\mathrm{A}} = p_{\mathrm{B}} + n_{\mathrm{B}}$ and $p_{\mathrm{A}} > p_{\mathrm{B}}$. In each atom of body A, working from the outermost electron shell inward, perform reverse beta decay by stuffing $p_{\mathrm{A}} - p_{\mathrm{B}}$ of its electrons, along with as many antineutrinos, into its nuclear protons, thus turning the protons into neutrons and the A atoms into B atoms, maintaining congruence all the while. Now the bodies are identically constituted and their weights, therefore their passive–inertial masses, are still the same. But if neither leptons nor binding energies generate gravity,[3] then the active mass of body A before the transformation is the same as that after the transformation, thus the same as that of B, and therefore the ratio $M_a/M_{p,i}$ is the same for A and B — despite that the passive–inertial masses and the binding energies of A's atoms and molecules have changed. It is therefore the case that a perfect-precision null result of the idealized Kreuzer experiment, and by straightforward extension the actual experiment, cannot rule out that leptons (and, concomitantly, binding energies) do not generate gravity.

It is conceivable that, by themselves, the changes in the passive–inertial masses and the binding energies of A's atoms and molecules would have increased A's active gravitational mass, but that this increase was exactly matched by a decrease owed to a change of molecular kinetic energy necessary to maintain A's size, shape, and weight. It is also conceivable that they would have *de*creased A's active mass, and that this *de*crease was compensated by a change of kinetic energy. It is, however, equally conceivable (and from a probabilistic standpoint even more likely) that none of these changes would cause any change in A's active gravitational mass. Consequently, just as a null result of the Kreuzer experiment cannot rule out nongravitating leptons, neither can it exclude that binding energy and kinetic energy do not produce gravity.

A formulation of this conclusion that makes no reference to transmutation of elements reads as follows: If

a. two homogeneous, geometrically congruent, electrically neutral, material bodies of equal densities have the same total number of protons and neutrons, and
b. every proton and every neutron, standing alone, would exhibit the same active gravitational mass as every other proton and every other neutron, and
c. no constituent, material or otherwise, of either body other than its protons and neutrons generates any gravitational effect at a point an experimentally reasonable distance from that body, and

[3] In the case of binding energies the 'ungenerated gravity' in question is the gravity outside the bodies, where the electromagnetic field vanishes. Within the bodies the electromagnetic field can be nonzero, thus might generate internal gravity not detectable externally. This interpretation will be maintained throughout the present Letter.

d. whatever nonlinearities exist in the superposition of the gravity of the protons and neutrons of either body approximate those of the other body no less closely than do the nonlinearities in the superpositions of the electromagnetic fields generated by the bodies' constituents,

then probing of the gravitational field at an experimentally reasonable distance from either of those bodies would yield no information that would allow one to decide which of the bodies was generating that field.

It is not simply that the Kreuzer experiment cannot rule out that leptons, binding energy, and kinetic energy do not gravitate. Rather it is that such nongravitating is fully consistent with absolute, precise constancy of the ratio of active to passive–inertial gravitational mass across all material bodies composed of atoms and molecules with protons and shell electrons in equal numbers, thus electrically neutral. For this reason the other experiments cited in [1] as capable, with improvements in precision, of demonstrating that leptons gravitate cannot do so, as the most they can do is increase confidence in the constancy of that ratio. What an experiment using material bodies carrying excess electric charge might show is, of course, a different matter.

The notion that energy in all its forms produces gravity traces all the way back to Einstein's 1916 paper Die Grundlage der allgemeinen Relativitätstheorie [3]. In that paper's §16, titled in translation The General Form of the Field Equations of Gravitation, Einstein seeks a tensorial equation to correspond to the Poisson equation $\nabla^2\phi = 4\pi\kappa\rho$, where ρ denotes the "density of matter". Drawing on the special theory of relativity's identification of "inert mass" with "energy, which finds its complete mathematical expression in . . . the energy-tensor", he concludes that "we must introduce a corresponding energy-tensor of matter T^{α}_{σ}". Further describing this energy-tensor as "corresponding to the density ρ in Poisson's equation", he goes on to invent the field equation that bears his name: $R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = -8\pi\kappa T_{\mu\nu}$, as currently expressed. Here Einstein confounded 'gravitat*ing* mass', which is the sole contributor to the "density of matter" in Poisson's equation, with "inert mass", thus with energy by way of $E = mc^2$ and with 'gravitat*ed*' mass by way of the equivalence between inertial mass and passive gravitational mass. Whether such a confounding can be justified by experimental evidence is the underlying question addressed by the previous Letter [1] and this one.[4]

[4] That Einstein confounded active mass with passive–inertial mass, knowingly or unknowingly, is borne out further by the statement in his §16 that for a "complete system (e.g. the solar system), the total mass of the system, and therefore its total gravitat*ing* action as well, will depend on the total energy of the system, and therefore on the *ponderable* energy together with the gravitational energy." (Emphases added.) Let it be remembered, however, that he thought of the field equation with the "energy-tensor of matter" in it as similar to a building with one wing made of fine marble (geometry) and the other of low-grade wood (energy-tensor), which ultimately should be replaced by an equation whose architectural analog would consist of marble alone [4].

If confounding of active gravitational mass with passive–inertial mass and with energy is not justified, then Einstein's field equation is open to modification in two ways: a) some of the forms of energy usually included in the tensor $T_{\mu\nu}$ can be left out; b) the couplings to geometry of those forms of energy left in can differ from the usual. In a paper that appeared some thirty years ago [5], exercise of option (b) to reverse the polarity of the coupling of a scalar field to geometry produced a space-time manifold which was described in [5] as a 'drainhole', and which has since been recognized as an early, perhaps the first example of what is now called a 'traversable wormhole' [6,7]. Specifically, with ϕ governed by the wave equation $\Box\phi := \phi^{,\kappa}{}_{;\kappa} = 0$ and scaled so that $T_{\mu\nu} = (1/4\pi\kappa)(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\phi^{,\kappa}\phi_{,\kappa}\, g_{\mu\nu})$, Einstein's equation was replaced by $R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = 8\pi\kappa T_{\mu\nu}$. Conventional wisdom says that this coupling somehow makes the energy of the scalar field be *negative*, and that the scalar field must therefore be associated with 'exotic' matter. But if, as argued here, the coupling of energy to geometry is not dictated by experimental observation, then one can just as well say that the energy of the scalar field is positive, that the reversed-polarity coupling is as justifiable as the conventional coupling, and that nothing 'exotic' is involved. This is clearly apparent in [5], where the gravitational field is untangled from the geometry of space, and the scalar field is seen to be coupled essentially to the geometry of space alone, even to the extent that gravity can be turned off completely while the drainhole stays open. The effect of the reversed polarity of the coupling is to let in the negative spatial curvatures that must be present if a stable traversable wormhole throat is to exist. With or without gravity turned on, the space-time is horizonless, geodesically complete, and singularity-free, the Penrose–Hawking singularity theorems [8] having been escaped by denial of their primary hypotheses.

A further demonstration of the reasonableness of the reversed-polarity coupling of the scalar field to geometry occurs in [9], which extracts from the field equations a metric describing a nonstatic, nongravitating drainhole–traversable-wormhole whose throat, starting with infinite radius in the infinitely distant past, chokes down to a single point, instantly reopens, then expands back to infinite size in the infinitely distant future.

Apropos of option (b), a relatively recent paper [10] has argued that not all types of energy are equivalent to mass, and that those that are not, such as electromagnetic energy, can couple to geometry in ways different from the way that mass couples to it.

As to option (a), if the presumption that kinetic energy generates gravity is not justified, then the same should be true for pressure in a fluid or a gas. This allows the usual mass-energy-stress-momentum tensor to be replaced by one without pressure terms. That produces a solution analogous to but simpler than the Schwarzschild interior solution, which I shall describe in a subsequent paper.

Lastly I would for historical purposes point out that, as noted in the previous Letter, the question whether electrons generate gravity was first posed explicitly twenty years ago [11], if not earlier.

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