MIDTERM 3 PRACTICE PROBLEMS

(12.4.1) Evaluate $\iint\limits_D e^{-x^2-y^2}\,dA$, where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis.

(12.5.1) Find the mass and center of mass of the lamina that occupies the region

$$D = \{(x, y) \mid 0 \le x \le 2, -1 \le y \le 1\}$$

and that has density function $\rho(x, y) = xy^2$.

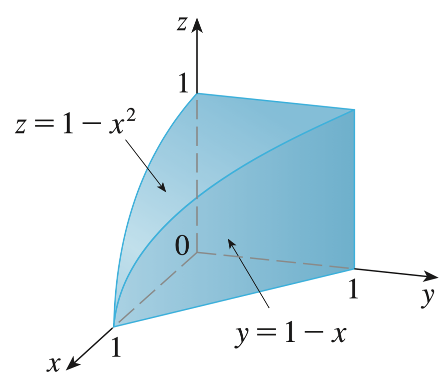
(12.6.1) Find the area of the part of the plane $x+2y+3z = 1$ that lies inside the cylinder $x^2+y^2 = 3$.

(12.6.2) Find the area of the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the plane $y = x$ and the parabolic cylinder $y = x^2$.

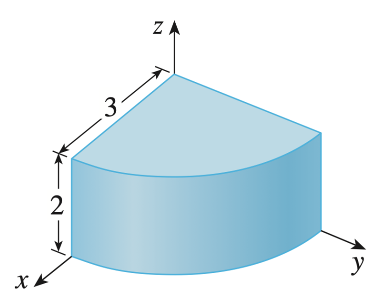
(12.7.1) The figure shows the region of integration for the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)\, dy\, dz\, dx$$

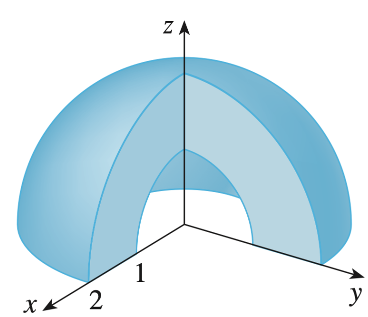
Rewrite this integral as an equivalent iterated integral in the five other orders.



(12.8.1) Set up the triple integral of an arbitrary continuous function $f(x, y, z)$ in cylindrical or spherical coordinates over the solid shown.



(a)



(b)

(12.9.1) Evaluate the integral $\iint\limits_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1,0),(2,0),(0,2)$ and $(0,1)$, by making an appropriate change of variables.

(13.1.1) Sketch the vector field $\mathbf{F}(x,y) = \langle \frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \rangle$.

(13.2.1) Evaluate the line integral $\int_C y^3\,ds$, where C is given by $\vec{r}(t) = \langle t^3, t\rangle$, for $0 \le t \le 2$.

(13.2.2) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = xy\,\mathbf{i} + 3y^2\,\mathbf{j}$ and C is parametrized by $\vec{r}(t) = \langle 11t^4, t^3\rangle$, for $0 \le t \le 1$.

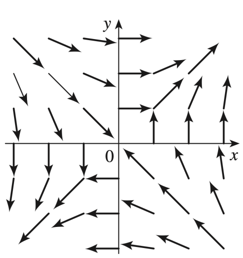
(13.3.1) Let $f(x,y) = 2x^2 + y^2$. Compute $\int_C \nabla f\cdot d\mathbf{r}$, where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

Answers

(12.4.1) $(\pi/2)(1-e^{-4})$
(12.5.1) $4/3, (\frac{4}{3}, 0)$
(12.6.1) $\sqrt{14}\pi$
(12.6.2) $\sqrt{2}/6$
(12.7.1)

$$\begin{aligned}
\int_0^1\int_0^{1-x^2}\int_0^{1-x} f(x,y,z)\,dy\,dz\,dx &= \int_0^1\int_0^{\sqrt{1-z}}\int_0^{1-x} f(x,y,z)\,dy\,dx\,dz \\
&= \int_0^1\int_0^{1-x}\int_0^{1-x^2} f(x,y,z)\,dz\,dy\,dx \\
&= \int_0^1\int_0^{1-y}\int_0^{1-x^2} f(x,y,z)\,dz\,dx\,dy \\
&= \int_0^1\int_{2y-y^2}^{1}\int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dz\,dy + \int_0^1\int_0^{2y-y^2}\int_0^{1-y} f(x,y,z)\,dx\,dz\,dy \\
&= \int_0^1\int_0^{1-\sqrt{1-z}}\int_0^{\sqrt{1-z}} f(x,y,z)\,dx\,dy\,dz + \int_0^1\int_{1-\sqrt{1-z}}^{1}\int_0^{1-y} f(x,y,z)\,dx\,dy\,dz
\end{aligned}$$

(12.8.1) (a) $\int_0^{\pi/2}\int_0^3\int_0^2 f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$
(12.9.1) $\frac{3}{2}\sin(1)$
(13.1.1) Sketch:



(13.2.1) $\frac{1}{54}(145^{3/2}-1)$
(12.2.1) 45
(12.3.1) 3