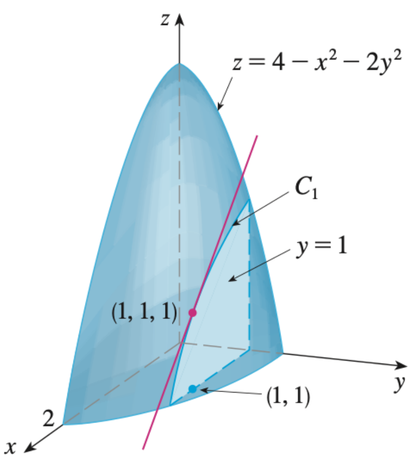
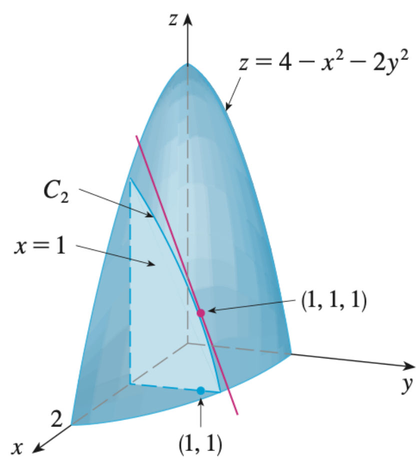
MIDTERM 2 PRACTICE PROBLEMS

(11.2.1) Find the limit of $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$, if it exists, or show that the limit does not exist.

(11.2.2) Find the limit of $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$, if it exists, or show that the limit does not exist.

(11.3.1) Using the graph of $z = 4 - x^2 - 2y^2$, is $f_x(1,1)$ positive or negative? Is $f_y(1,1)$ positive or negative? Check your answer by computing $f_x(1,1)$ and $f_y(1,1)$.





(11.4.1) Find an equation of the tangent plane to $z = y\cos(x-y)$ at the point $(2,2,2)$.

(11.5.1) If $z = f(x,y)$, where f is differentiable, and $x = g(t), y = h(t)$, $g(3) = 2$, $g'(3) = 5, h(3) = 7, h'(3) = -4, f_x(2,7) = 6, f_y(2,7) = -8$, find dz/dt when $t = 3$.

(11.5.2) Let $R = \ln(u^2+v^2+w^2)$ and $u = x+2y, v = 2x-y, w = 2xy$. Find $\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$ when $x = y = 1$.

(11.6.1) Find the directional derivative of $f(x,y) = ye^{-x}$ at the point $(0,4)$ in the direeection of $\theta = 2\pi/3$.

(11.6.2) Suppose that over a certain region of space, the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.

(a) Find the rate of change of the potential at $P(3,4,5)$ in the direction of the vector $\vec{v} = \vec{i} + \vec{j} - \vec{k}$.
(b) In which direction does V change most rapidly at P?
(c) What is the maximum rate of change at P?

(11.6.3) Find an equation of the tangent plane to $z + 1 = xe^y\cos z$ at the point $(1,0,0)$.

(11.7.1) Find the absolute maximum and minimum values of $f(x, y) = 1 + 4x - 5y$ on the set D, where D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$.

(11.8.1) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $x^2 + 2y^2 = 6$.

(11.8.2) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 1$.

(12.2.1) Calculate the iterated integral $\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy$.

(12.2.2) Calculate the iterated integral $\int_0^2 \int_0^{\pi/2} (x \sin y)\, dy\, dx$.

(12.3.1) Express D as a type I region and also as a type II region. Then evaluate the double integral in two ways.
(a) $\iint_D x\, dA$, D is enclosed by the lines $y = x, y = 0, x = 1$.
(b) $\iint_D xy\, dA$, D is enclosed by the curves $y = x^2, y = 3x$.

Answers

(11.2.1) Does not exist. (Consider the lines $y = mx$)
(11.2.2) 0 (Use polar coordinates)
(11.3.1) $f_x(1, 1) = -2$ and $f_y(1, 1) = -4$.
(11.4.1) $z = y$
(11.5.1) 62
(11.5.2) $\frac{9}{7}, \frac{9}{7}$
(11.6.1) $2 + \sqrt{3}/2$
(11.6.2) (a) $32/\sqrt{3}$, (b) $\langle 38, 6, 12\rangle$, (c) $2\sqrt{406}$.
(11.6.3) $x + y - z = 1$
(11.7.1) Maximum $f(2, 0) = 9$, minimum $f(0, 3) = -14$.
(11.8.1) Maximum $f(\pm 2, 1) = 4$, minimum $f(\pm 2, -1) = -4$.
(11.8.2) Maximum $f(\pm\sqrt{2/3}, 1/\sqrt{3}) = 2/(3\sqrt{3})$, minimum $f(\pm\sqrt{2/3}, -1/\sqrt{3}) = -2/(3\sqrt{3})$
(12.2.1) 10
(12.2.2) 2
(12.3.1a) Type I: $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le x\}$. Type II: $D = \{(x, y) \mid 0 \le y \le 1, y \le x \le 1\}$. $\iint_D x\, dA = \frac{1}{3}$.