MIDTERM 1 PRACTICE PROBLEMS

Section 9.2

(1) Copy the vectors in the figure and use them to draw the following vectors.

(a) $\vec{a} + \vec{b}$

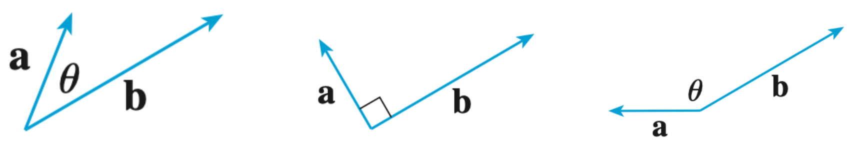
(b) $\vec{a} - \vec{b}$

(c) $2\vec{b} - \vec{a}$

(2) Find a unit vector that has the same direction as $8\vec{i} - \vec{j} + 4\vec{k}$.

Section 9.3

(1) In each case, state whether $\vec{a} \cdot \vec{b}$ is positive, negative, or zero. Explain your reasoning.



(2) Find the angle between the vectors $\langle -8, 6\rangle$ and $\langle \sqrt{7}, 3\rangle$.

(3) Find the work done by a force $F = 8\vec{i} - 6\vec{j} + 9\vec{k}$ that moves an object from $(0, 10, 8)$ to $(6, 12, 20)$ along a straight line. Distances in meters, force in newtons.

(4) Find the scalar and vector projections of $\vec{b} = \langle 0, 1, \frac{1}{2}\rangle$ onto $\vec{a} = \langle 2, -1, 4\rangle$.

Section 9.4

(1) Find the cross product $\langle 1, 3, -2\rangle \times \langle -1, 0, 5\rangle$.

Section 9.5

(1) Find a vector equation, parametric equations, and symmetric equations for the line through $(6, 1, -3)$ and $(2, 4, 5)$.

(2) Know how to write down the equation of a plane:

- Using a point and a normal vector. Example: plane through $(6, 3, 2)$ perpendicular to $\langle -2, 1, 5\rangle$.
- Using three non-collinear points. Example: $(0, 1, 1)$, $(1, 0, 1)$, $(1, 1, 0)$.
- Using two intersecting lines. Example: $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-5}{6}$ and $\vec{r}(t) = \langle 1, -1, 5\rangle + t\langle 1, 1, -3\rangle$.
- Using a point and a line. Example: through $(6, 0, 2)$ and containing the line $x = 4 - 2t,\ y = 3 + 5t,\ z = 7 + 4t$.

(3) Find the distance from $(1, -2, 4)$ to the plane $3x + 2y + 6z = 5$.

Section 9.7

(1) Identify the surface whose equation is given.
- $\theta = \pi/4$
- $\phi = \pi/3$
- $z = 4 - r^2$
- $r = 2\cos\theta$
- $\rho = \sin\theta \sin\phi$

Section 10.1

(1) Find a vector function that represents the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.
(2) Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

Section 10.2

(1) Let $\vec{r}(t) = \cos t\,\vec{i} + 3t\,\vec{j} + 2\sin(2t)\,\vec{k}$. Find the unit tangent vector $\vec{T}(t)$ at $t = 0$.
(2) Suppose $\vec{v}(t) = 16t^3\vec{i} - 9t^2\vec{j} + 25t^4\vec{k}$. If the initial position is $\langle 1, 4, 3\rangle$ at $t = 0$, find the position at $t = 1$.

Section 10.3

(1) Find the length of $\vec{r}(t) = \langle 2\sin t,\, 5t,\, 2\cos t\rangle$ for $-10 \le t \le 10$.
(2) Write an integral that computes the length of $\vec{r}(t) = \langle \sqrt{t},\, t,\, t^2\rangle$ from $t = 1$ to $t = 4$.

Section 10.5

(1) Parametrize a cylinder of radius 3 centered on the y-axis.
(2) Parametrize a sphere of radius 2 centered at the origin.
(3) Parametrize the bottom hemisphere of a sphere with radius 10 centered at the origin.
(4) Parametrize the plane $2x - y + 3z = -2$.
(5) Parametrize the cone $4x^2 + 2y^2 = z^2$.
- Hint: Start with $u^2 + v^2 = z^2$. Then set $u = 2x$ and $v = \sqrt{2}\,y$.

(6) Parametrize the ellipsoid $3x^2 + 5y^2 + z^2 = 1$.
- Hint: Start with $u^2 + v^2 + z^2 = 1$. Then set $u = \sqrt{3}\,x$ and $v = \sqrt{5}\,y$.