Chapter 10

10.1 Vector Functions and Space Curves

- What is a vector-valued function? What are the component functions of a vector-valued function?
- Let $\vec{r}(t) = \langle t^3, \ln(3-t), \sqrt{t}\rangle$. What are the component functions? What is the domain of $\vec{r}(t)$?
- How to take a limit of a vector-valued function?
- Find $\lim_{t\to 0} \vec{r}(t)$, where $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \frac{\sin t}{t}\vec{k}$.
- What does it mean for a vector function $\vec{r}(t)$ to be continuous at a?
- What is a space curve?
- Describe the curve defined by the vector function $\vec{r}(t) = \langle 1+t, 2+5t, -1+6t\rangle$.
- Plane curves can also be represented in vector notation. How would we write the curve given by the parametric equations $x = t^2 - 2t$ and $y = t+1$ in vector notation?
- Sketch the curve whose vector equation is $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$.
- Find a vector equation for the line segment that joins the point $P(1,3,-2)$ to the point $Q(2,-1,3)$.
- Find a vector function that represents the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $y+z=2$.

10.2 Derivatives and Integrals of Vector Functions

- What is the derivative of a vector function $\vec{r}(t)$?
- Explain the derivative of a vector function geometrically.
- What is the unit tangent vector?
- If $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$, where f, g, and h are differentiable functions, show that $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t)\rangle$.
- Find the derivative of $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \sin 2t\vec{k}$. Find the unit tangent vector at the point where $t = 0$.
- For the curve $\vec{r}(t) = \sqrt{t}\vec{i} + (2-t)\vec{j}$, find $\vec{r}\,'(t)$ and sketch the position vector $\vec{r}(1)$ and the tangent vector $\vec{r}\,'(1)$.
- Find parametric equations for the tangent line to the helix with parametric equations

$$x = 2\cos t \qquad y = \sin t \qquad z = t$$

  at the point $(0, 1, \pi/2)$.
- What is the second derivative of a vector function $\vec{r}$? What is the second derivative of the function in the previous example?
- Suppose $\vec{u}$ and $\vec{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Find differentation formulas for the following expressions.

  1. $\dfrac{d}{dt}[\vec{u}(t) + \vec{v}(t)] =$
  2. $\dfrac{d}{dt}[c\vec{u}(t)] =$
  3. $\dfrac{d}{dt}[f(t)\vec{u}(t)] =$
  4. $\dfrac{d}{dt}[\vec{u}(t) \cdot \vec{v}(t)] =$
  5. $\dfrac{d}{dt}[\vec{u}(t) \times \vec{v}(t)] =$
  6. $\dfrac{d}{dt}[\vec{u}(f(t))] =$

- Show that if $\vec{r}(t)$ has constant length, then $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal for all t. What does this result mean geometrically?
- What is the definite integral of a continuous vector function $\vec{r}(t)$?
- How can we extend the Fundamental Theorem of Calculus to continuous vector functions?
- Find $\int_0^{\pi/2} \vec{r}(t)\,dt$ if $\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j} + 2t\vec{k}$.

10.3 Arc Length

- How do we find the length of a plane curve with parametric equations $x = f(t), y = g(t)$ for $a \leq t \leq b$?
- How do we find the length of a space curve?
- How can we write the arc length formula more compactly?
- Find the length of the arc of the circular helix with vector equation $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.
- Can a single curve C be represented by more than one vector function? What is a parametrization? Does arc length depend on the parametrization of C?
- Suppose that C is a curve given by a vector function

$$\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k} \qquad a \leq t \leq b$$

  where $\vec{r}\,'$ is continuous and C is traversed exactly once as t increases from a to b. What is the arc length function $s(t)$?
- Why is it often useful to parametrize a curve with respect to arc length?
- Reparametrize the helix $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing t.

10.5 Parametric Surfaces

- What is a parametric surface?
- Identify and sketch the surface with vector equation

$$\vec{r}(u, v) = 2\cos u\vec{i} + v\vec{j} + 2\sin u\vec{k}$$

- How can we modify the previous example to obtain a quarter-cylinder with length 3?
- What are grid curves?
- On the graph of the surface

$$\vec{r}(u, v) = \langle (2 + \sin v)\cos u, (2 + \sin v)\sin u, u + \cos v \rangle$$

  which grid curves have u constant? Which have v constant?
- Find a vector function that represents the plane that passes through the point P_0 with position vector r_0 and that contains two non-parallel vectors $\vec{a}$ and $\vec{b}$.
- Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$. What are the grid curves?
- Find a parametric representation for the cylinder

$$x^2 + y^2 = 4 \qquad 0 \leq z \leq 1$$

- Find a vector function that represents the elliptic paraboloid $z = x^2 + 2y^2$.
- Suppose a surface S is given as the graph of a function of x and y, that is, with an equation of the form $z = f(x, y)$. How can we view S as a parametric surface?
- Find a parametric representation for the top half of the cone $z^2 = 4x^2 + 4y^2$.
- What is a surface of revolution? How can we represent a surface of revolution parametrically?
- Find parametric equations for the surface generated by rotating the curve $y = \sin x, 0 \leq x \leq 2\pi$ about the x-axis. Use these equations to graph the surface of revolution.