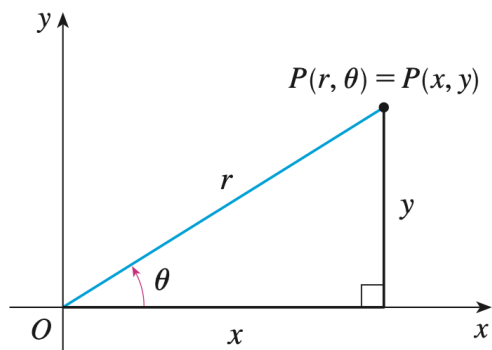
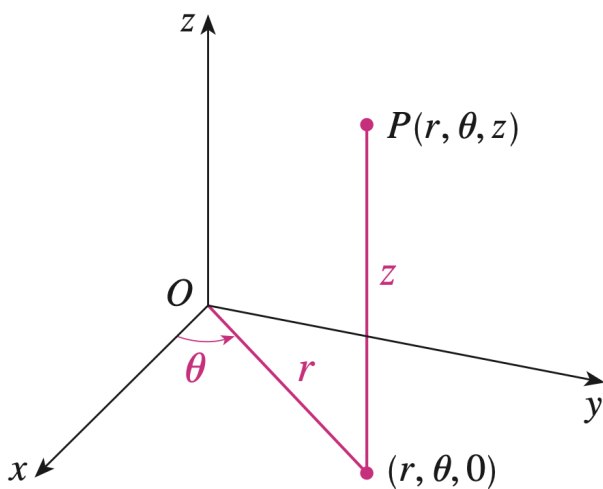


9.7 Cylindrical and Spherical Coordinates

Question. In two dimensions, how do we convert between polar and Cartesian coordinates?



Definition. What is the cylindrical coordinate system?



Example.

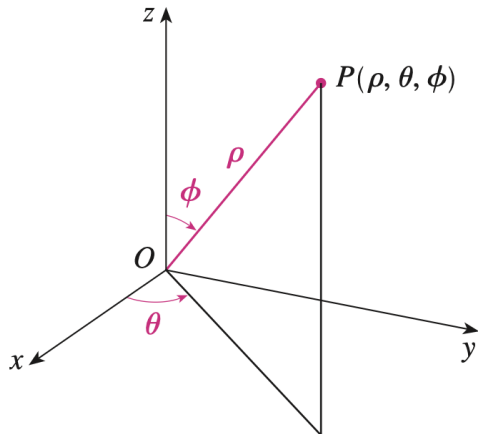
- (a) Plot the point with cylindrical coordinates $(2, 2\pi/3, 1)$ and find its rectangular coordinates.
- (b) Find cylindrical coordinates of the point with rectangular coordinates $(3, -3, -7)$.

Remark. When are cylindrical coordinates useful? What is the equation of a cylinder in cylindrical coordinates?

Example. Describe the surface whose equation in cylindrical coordinates is $z = r$

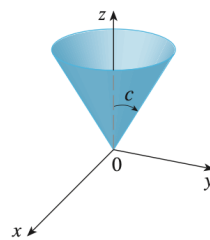
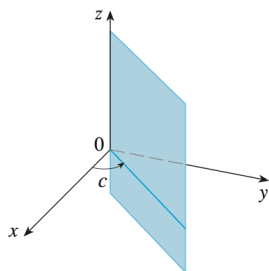
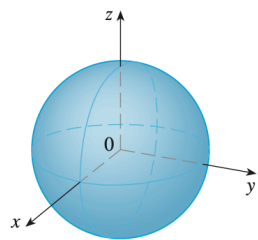
Example. Find an equation in cylindrical coordinates for the ellipsoid $4x^2 + 4y^2 + z^2 = 1$.

Definition. What is the spherical coordinate system?

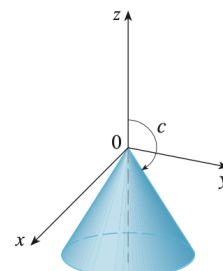


Remark. When are spherical coordinates useful? What is the equation of a sphere in spherical coordinates?

Example. Find an equation for each of the following surfaces using spherical coordinates.

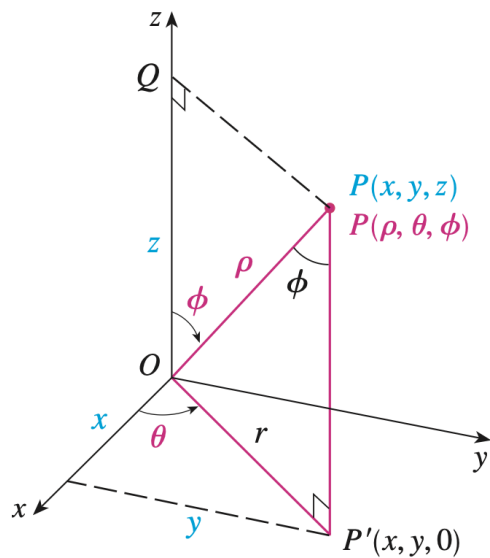


$$0 < c < \pi/2$$

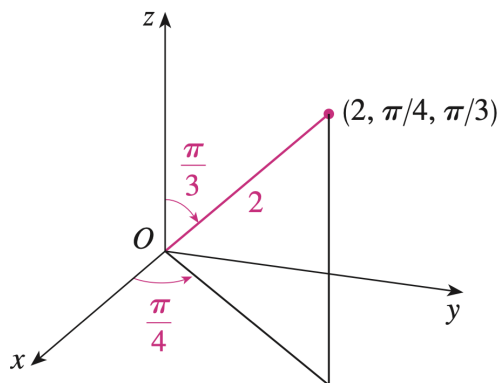


$$\pi/2 < c < \pi$$

Question. What is the relationship between rectangular and spherical coordinates?



Example. The point $(2, \pi/4, \pi/3)$ is given in spherical coordinates. Plot the point and find its rectangular coordinates.



Example. The point $(0, 2\sqrt{3}, -2)$ is given in rectangular coordinates. Find spherical coordinates for this point.

Example. Find an equation in spherical coordinates for the hyperboloid of two sheets with equation $x^2 - y^2 - z^2 = 1$.

Example. Find a rectangular equation for the surface whose spherical equation is $\rho = \sin \theta \sin \phi$.