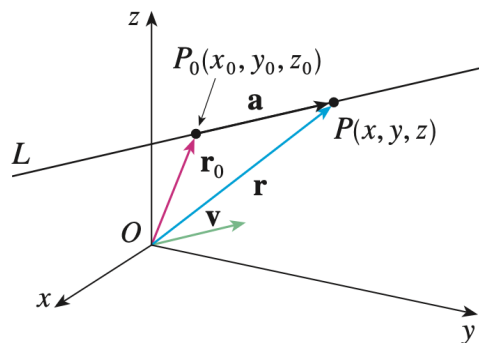
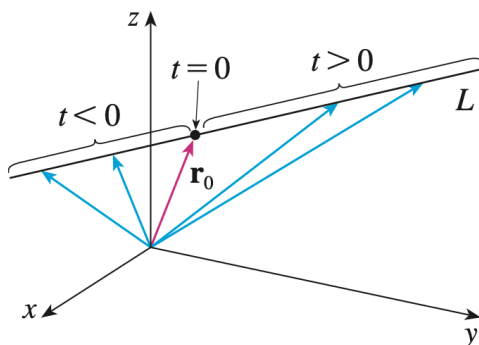


9.5 Equations of Lines and Planes

Question. How do we describe a line in three-dimensional space?



Question. How can we describe a line in three-dimensional space parametrically?



Example. Find a vector equation and parametric equations for the line that passes through the point $(5,1,3)$ and is parallel to the vector $\vec{i} + 4\vec{j} - 2\vec{k}$.

Question. Are the vector equation and parametric equations of a line unique?

Definition. What are the symmetric equations a line L ?

Example.

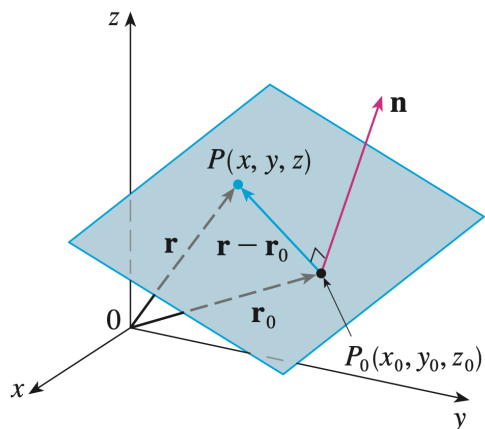
- (a) Find parametric equations and symmetric equations of the line that passes through the points $A(2, 4, -3)$ and $B(3, -1, 1)$.
- (b) At what point does this line intersect the xy -plane?

Example. Show that the lines L_1 and L_2 with parametric equations

$$\begin{array}{lll} x = 1 + t & y = -2 + 3t & z = 4 - t \\ x = 2s & y = 3 + s & z = -3 + 4s \end{array}$$

are skew lines.

Question. How to describe a plane in space?



Definition. How to describe a plane using a scalar equation?

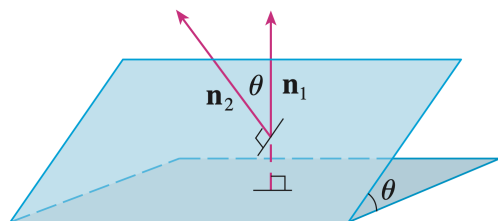
Example. Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\vec{n} = \langle 2, 3, 4 \rangle$.

Question. How to rewrite the equation of a plane using a linear equation?

Example. Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$ and $R(5, 2, 0)$.

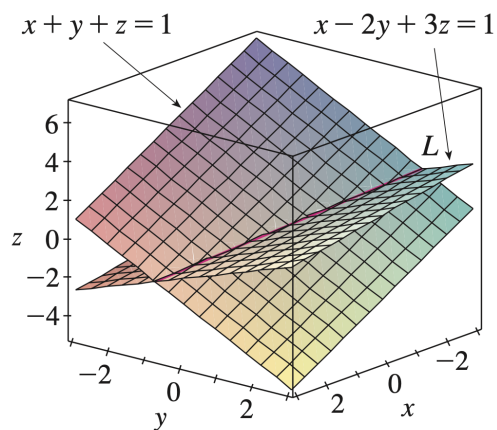
Example. Find the point at which the line with parametric equations $x = 2 + 3t$, $y = -4t$, $z = 5 + t$ intersects the plane $4x + 5y - 2z = 18$.

Definition. How can we determine if two planes are parallel? If two planes are not parallel, what is the angle between the two planes?

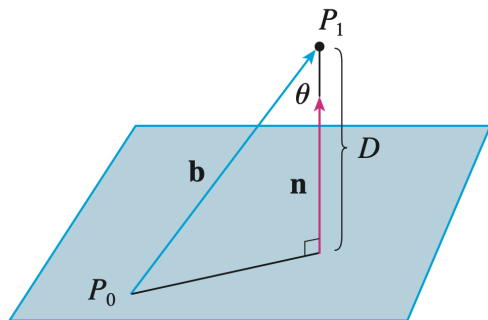


Example.

- Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.
- Find symmetric equations for the line of intersection L of these two planes.



Example. Find a formula for the distance D from a point $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$.



Example. Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$.

Example. We showed that the lines

$$\begin{array}{lll} L_1 : & x = 1 + t & y = -2 + 3t & z = 4 - t \\ L_2 : & x = 2s & y = 3 + s & z = -3 + 4s \end{array}$$

are skew. Find the distance between them.