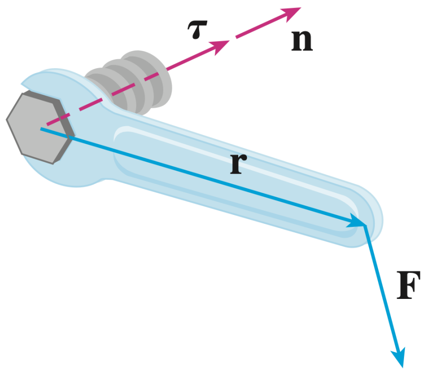
Lecture Notes
Math 2400 - Calculus III
Spring 2024

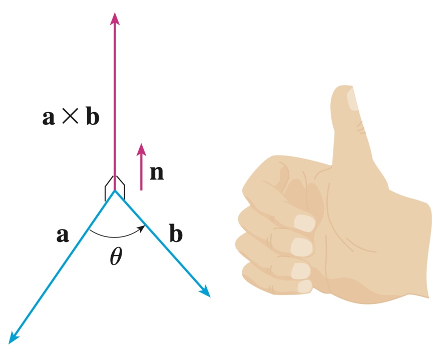
Name: ____________________________

## 9.4 The Cross Product

**Definition.** What is torque?

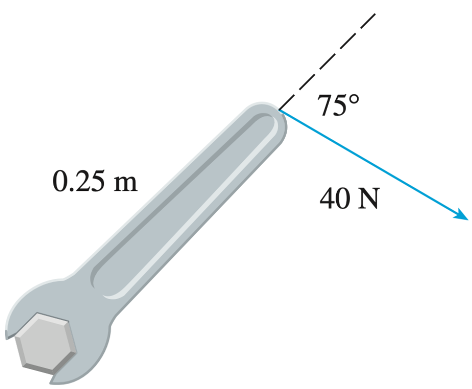



**Definition.** If $\vec{a}$ and $\vec{b}$ are nonzero vectors in $V_3$, what is the cross product of $\vec{a}$ and $\vec{b}$?



**Theorem.** Show that two nonvectors $\vec{a}$ and $\vec{b}$ are parallel if and only if $\vec{a} \times \vec{b} = 0$.

**Example.** A bolt is tightened by applying a 40-N force to a 0.25-m wrench, as shown below. Find the magnitude of the torque about the center of the bolt.



**Example.** Find $\vec{i} \times \vec{j}$ and $\vec{j} \times \vec{i}$.

**Example.** Summarize the cross products of $\vec{i}$, $\vec{j}$, and $\vec{k}$.

**Example.** Is $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ ?

**Example.** Is $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ ?

**Theorem.** What are some properties of the cross product? If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors and $c$ is a scalar, then

1. $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
2. $(c\vec{a}) \times \vec{b} = c(\vec{a} \times \vec{b}) = \vec{a} \times (c\vec{b})$
3. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
4. $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

**Question.** Give a geometric interpretation of the length of the cross product.

**Question.** How to calculate $\vec{a} \times \vec{b}$ using components?

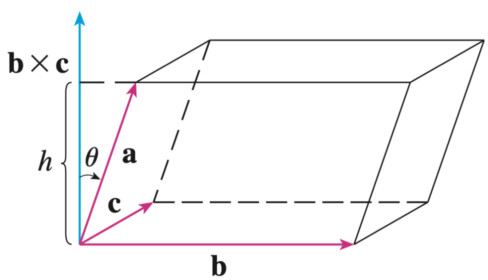
**Remark.** How can we use determinants to simplify the cross product calculation?

**Example.** Use determinants to calculate the cross product of $\vec{a} = \langle 1, 3, 4 \rangle$ and $\vec{b} = \langle 2, 7, -5 \rangle$.

**Example.** Find a vector perpendicular to the plane that passes through the points $P(1, 4, 6)$, $Q(-2, 5, -1)$, $R(1, -1, 1)$.

**Example.** Find the area of the triangle with vertices $P(1, 4, 6)$, $Q(-2, 5, -1)$, and $R(1, -1, 1)$.

**Definition.** What is the scalar triple product of the vectors $\vec{a}, \vec{b}$, and $\vec{c}$?



**Theorem.** Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$, and $\vec{c} = \langle c_1, c_2, c_3 \rangle$. Show that

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

**Example.** Use the scalar triple product to show that the vectors $\vec{a} = \langle 1, 4, -7 \rangle$, $\vec{b} = \langle 2, -1, 4 \rangle$, and $\vec{c} = \langle 0, -9, 18 \rangle$ lie in the same plane.