Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

9.3 The Dot Product

Definition. What is the work done by a force F in moving an object through a distance d?

Definition. What is the dot product of two nonzero vectors $\vec{a}$ and $\vec{b}$?

Example. How can we reinterpret work in terms of the dot product?

Example. A wagon is pulled a distance of 100 m along a horizontal path by a constant force of 70 N. The handle of the wagon is held at an angle of 35° above the horizontal. Find the work done by the force.

Definition. What does it mean for two vectors to be orthogonal?

Theorem. Prove that two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$.

Definition. What is the dot product in terms of components?

Example. Compute the following dot products:

(a) $\langle 2, 4\rangle \cdot \langle 3, -1\rangle$

(b) $\langle -1, 7, 4\rangle \cdot \langle 6, 2, -\frac{1}{2}\rangle$

(c) $(\vec{i} + 2\vec{j} - 3\vec{k}) \cdot (2\vec{j} - \vec{k})$

Example. Show that $2\vec{i} + 2\vec{j} - \vec{k}$ is perpendicular to $5\vec{i} - 4\vec{j} + 2\vec{k}$.

Example. Find the angle between the vectors $\vec{a} = \langle 2, 2, -1 \rangle$ and $\vec{b} = \langle 5, -3, 2 \rangle$.

Definition. What is the scalar projection of $\vec{b}$ onto $\vec{a}$?

Definition. What is the vector projection of $\vec{b}$ onto $\vec{a}$?

Example. Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -2, 3, 1 \rangle$.