Lecture Notes
Math 2400 - Calculus III
Spring 2024

Name: _____

9.2 Vectors

Definition. What is a vector?

Question. How to represent vectors?

Definition. Suppose a particle moves along a line segment from point A to point B. What is the displacement vector from A to B?

Definition. How do we add vectors?

Question. Given vectors $\vec{u}$ and $\vec{v}$, why is $\vec{u} + \vec{v} = \vec{v} + \vec{u}$?

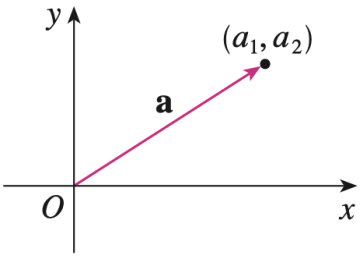
Example. Draw the sum of the vectors $\vec{a}$ and $\vec{b}$ shown below.

Definition. How do we scale vectors?

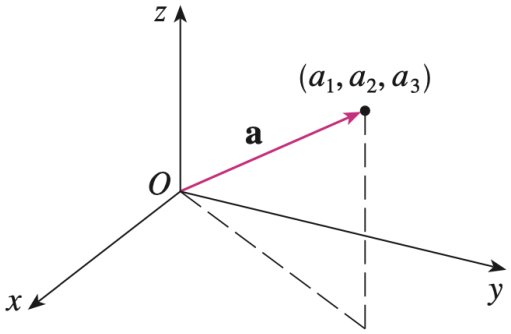
Definition. How do we subtract vectors?

Example. If $\vec{a}$ and $\vec{b}$ are the vectors shown below, draw $\vec{a} - 2\vec{b}$.

Question. How to represent vectors using coordinates?

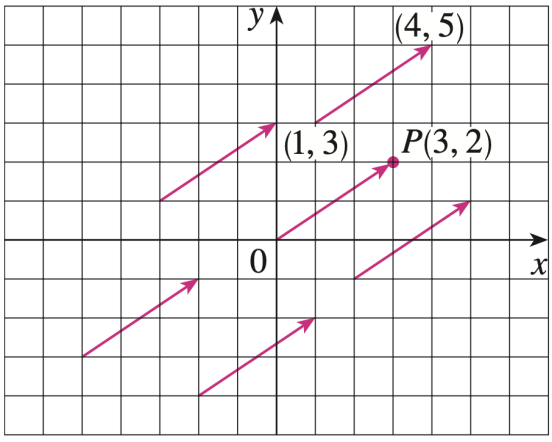


$\mathbf{a} = \langle a_1, a_2 \rangle$



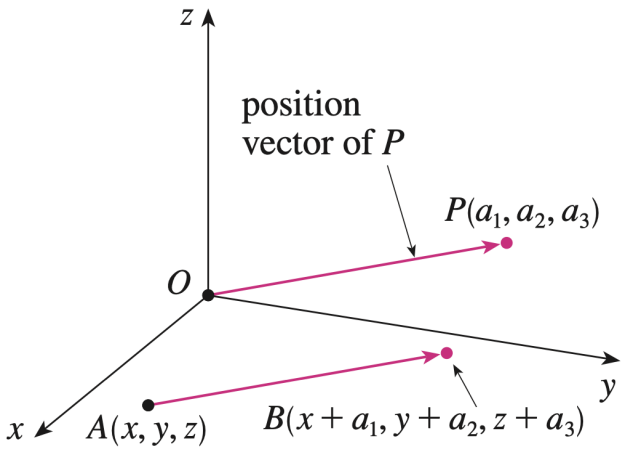
$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

Question. Explain why the vectors below are all equivalent.



Definition. What is a position vector?

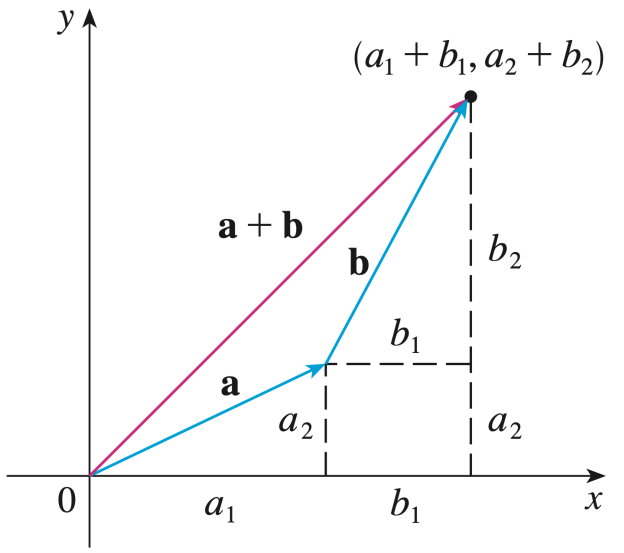
Definition. Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, what is the vector with representation $\vec{AB}$?



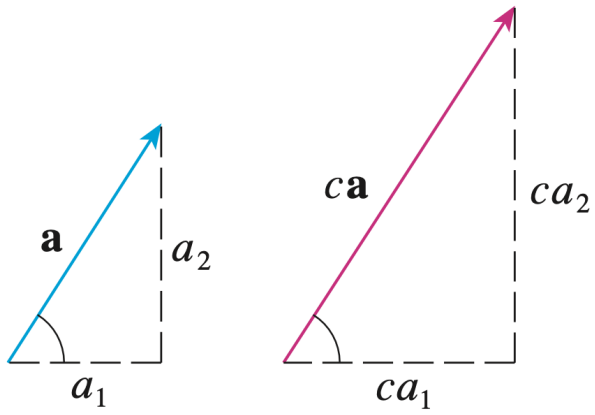
Example. Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$.

Definition. How to calculate the magnitude of a vector $\vec{v}$?

Question. How do we add vectors algebraically?



Question. How do we scale vectors algebraically?



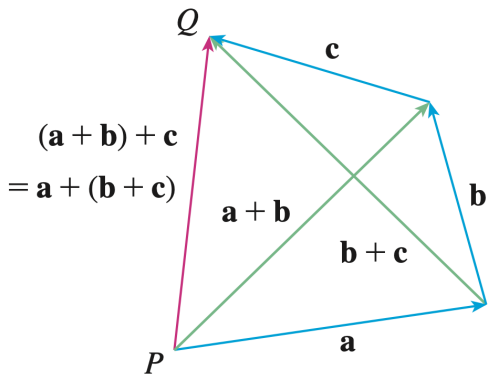
Example. If $\vec{a} = \langle 4, 0, 3 \rangle$ and $\vec{b} = \langle -2, 1, 5 \rangle$, find $|\vec{a}|$ and the vectors $\vec{a} + \vec{b}, \vec{a} - \vec{b}, 3\vec{b}$ and $2\vec{a} + 5\vec{b}$.

Definition. What is the set V_n?

Theorem. Let $\vec{a}, \vec{b}$, and $\vec{c}$ are vectors in V_n and c and d are scalars. Prove the following properties of these vectors.

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3. $\vec{a} + \vec{0} = \vec{a}$
4. $\vec{a} + (-\vec{a}) = \vec{0}$
5. $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
6. $(c + d)\vec{a} = c\vec{a} + d\vec{a}$
7. $(cd)\vec{a} = c(d\vec{a})$
8. $1 \cdot \vec{a} = \vec{a}$

Proof.



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Definition. What are the vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$?

Example. Show that any vector in V_3 can be expressed in terms of the vectors $\vec{i}$, $\vec{j}$, $\vec{k}$.

Example. If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$, express the vector $2\vec{a} + 3\vec{b}$ in terms of $\vec{i}, \vec{j}$, and $\vec{k}$.

Definition. What is a unit vector? If $\vec{a} \neq 0$, find a unit vector that has the same direction as $\vec{a}$.

Example. Find the unit vector in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$.

Example. A 100-lb weight hangs from two wires as shown below. Find the tensions (forces) $\vec{T_1}$ and $\vec{T_2}$ in both wires and the magnitudes of the tensions.