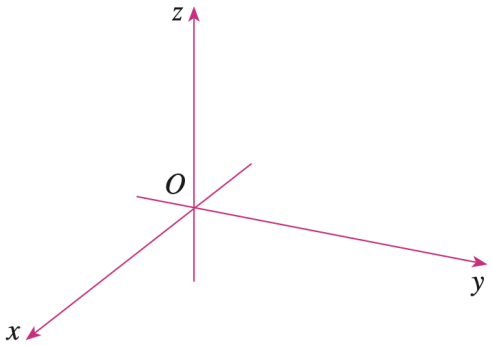
Lecture Notes
Math 2400 - Calculus III
Spring 2024

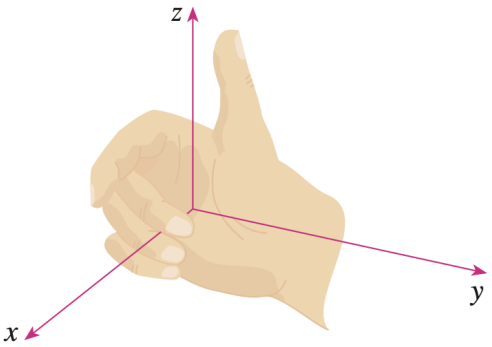
Name: _____

## 9.1 Three-Dimensional Coordinate Systems

**Question.** How do we represent points in space?

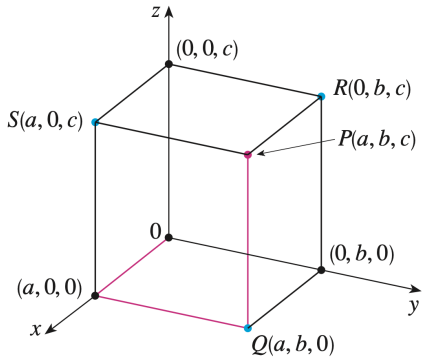



**Definition.** What is the right-hand rule?




**Example.** What are the three coordinate planes?

**Question.** Explain how a point $P(a, b, c)$ can be projected to each of the coordinate planes.




**Definition.** What is $\mathbb{R}^3$?

**Example.** What surface in $\mathbb{R}^3$ is represented by the equation $z = 3$?

**Example.** What surface in $\mathbb{R}^3$ is represented by the equation $y = 5$?

**Remark.** Why do we have to be careful about the context of our equations?

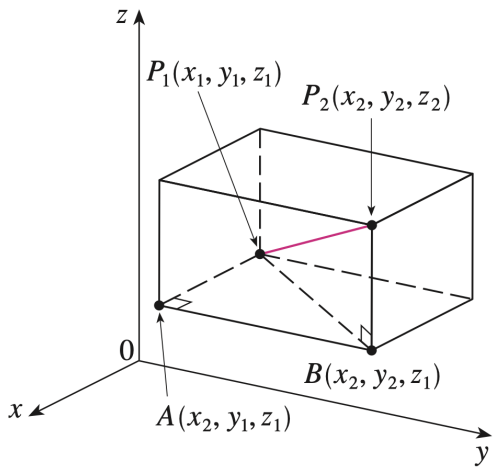
**Example.**

(a) Which points $(x, y, z)$ satisfy the equations $x^2 + y^2 = 1$ and $z = 3$?

(b) What does the equation $x^2 + y^2 = 1$ represent as a surface in $\mathbb{R}^3$?

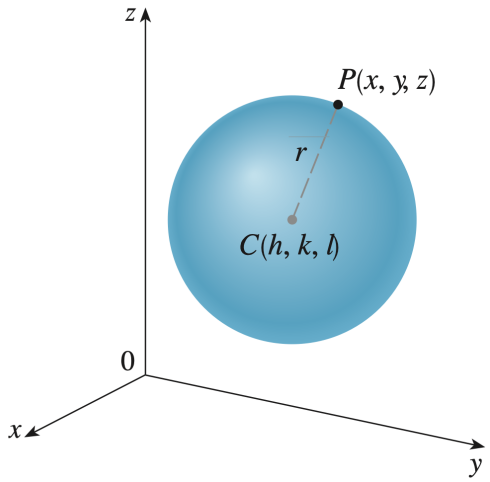
**Example.** Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $y = x$.

**Theorem** (Distance Formula in Three Dimensions)**.** Find a formula that represents the distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$.



**Example.** Find the distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$.

**Example.** Find an equation of a sphere with radius $r$ and center $C(h, k, l)$.



**Example.** Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere, and find its center and radius.

**Example.** What region in $\mathbb{R}^3$ is represented by the following inequalities?

$$1 \leq x^2 + y^2 + z^2 \leq 4 \qquad z \leq 0$$