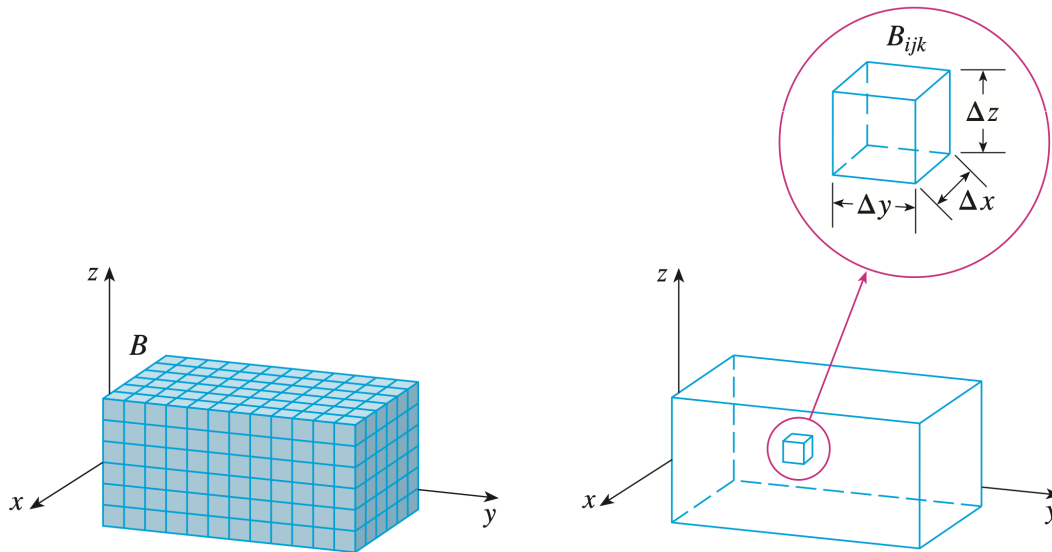


12.7 Triple Integrals

Definition. What is the triple integral of f over the box B ?

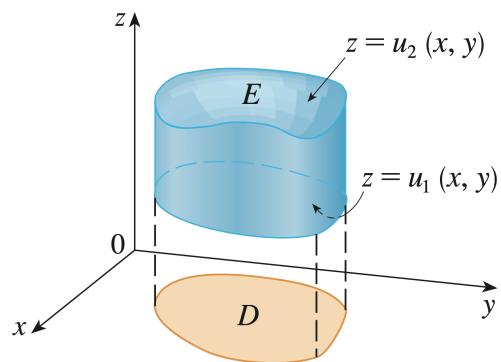


Theorem (Fubini's Theorem for Triple Integrals). If f is continuous on the rectangular box $B = [a, b] \times [c, d] \times [r, s]$, then how can we evaluate $\iiint_B f(x, y, z) dV$ using an iterated integral?

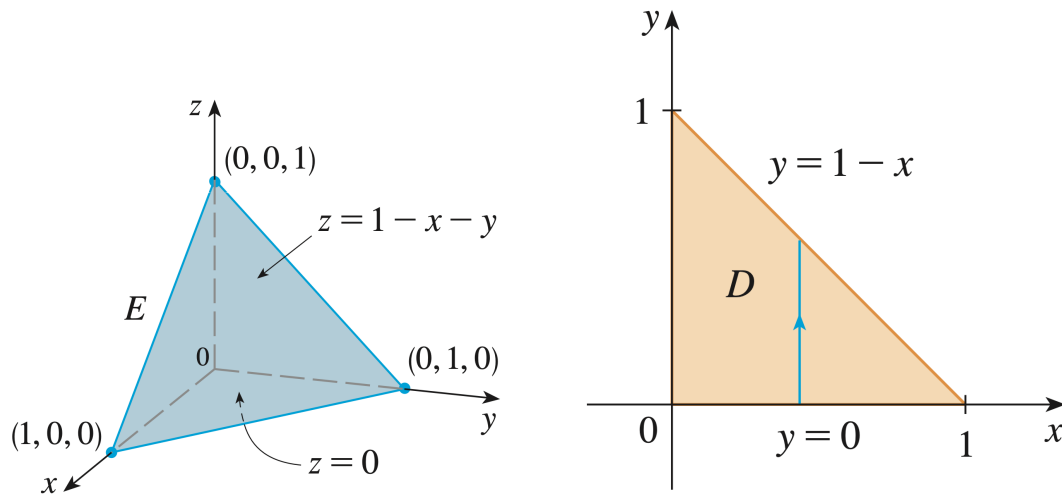
Example. Evaluate the triple integral $\iiint_B xyz^2 dV$, where B is the rectangular box given by

$$B = \{(x, y, z) \mid 0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$$

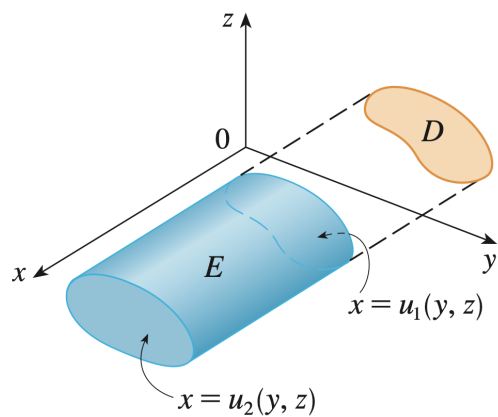
Definition. What is a type 1 solid region? If E is type 1, what is $\iiint_E f(x, y, z) dV$?



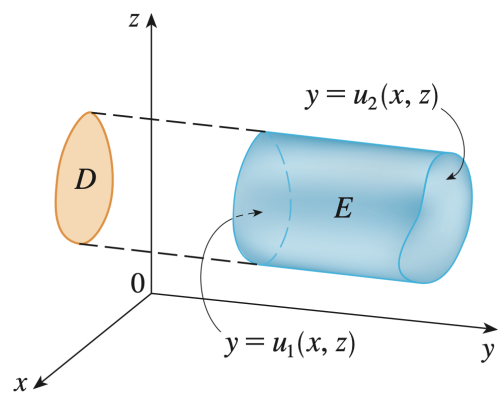
Example. Evaluate $\iiint_E z \, dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.



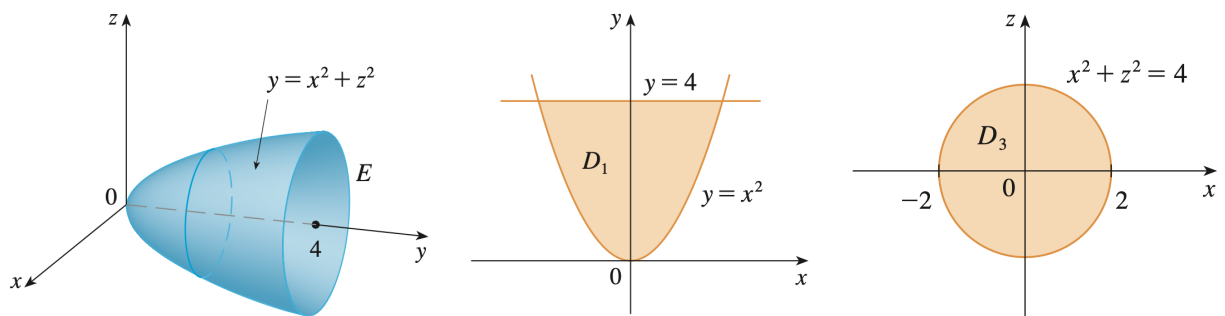
Definition. What is a type 2 solid region? If E is type 2, what is $\iiint_E f(x, y, z) dV$?



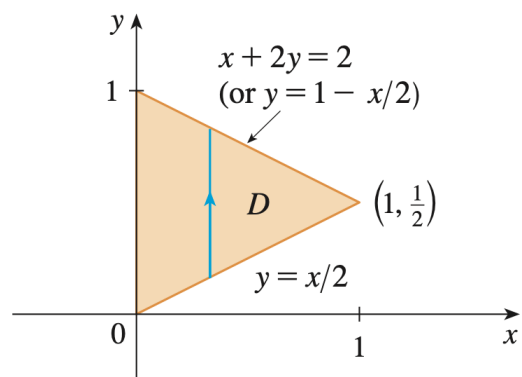
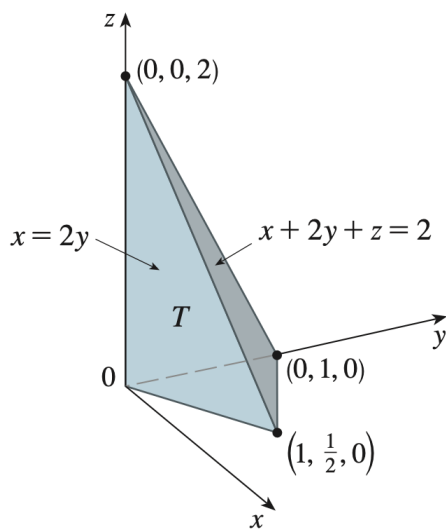
Definition. What is a type 3 solid region? If E is type 3, what is $\iiint_E f(x, y, z) dV$?



Example. Evaluate $\iiint_E \sqrt{x^2 + z^2} \, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.



Example. Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.



Definition. How can we compute the mass and the center of mass of a solid object that occupies the region E if its density, in units of mass per unit volume, at any given point (x, y, z) is given by $\rho(x, y, z)$?

Example. Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$, and $x = 1$.

