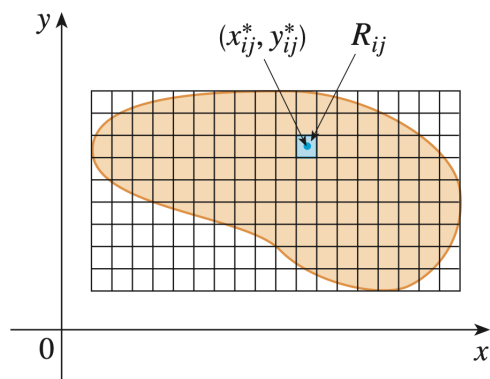
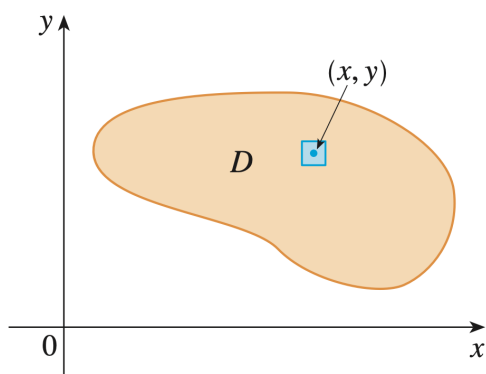
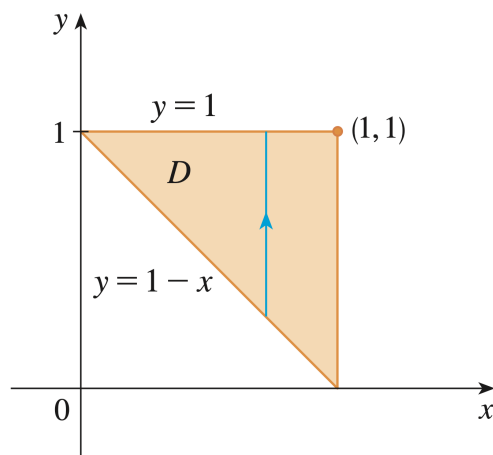


12.5 Applications of Double Integrals

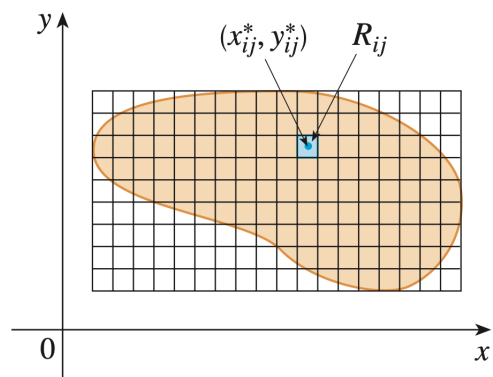
Question. Suppose a lamina (a thin plate) occupies a region D , and its density at a point (x, y) is given by $\rho(x, y)$. How can we compute the total mass of the lamina?



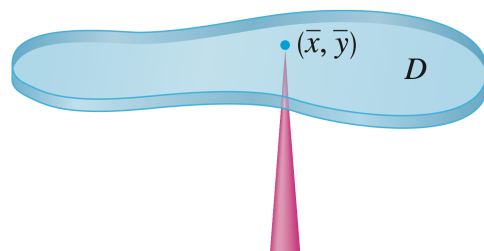
Example. Charge is distributed over the triangular region D so that the charge density at (x, y) is $\sigma(x, y) = xy$, measured in coulombs per square meter (C/m^2). Find the total charge.



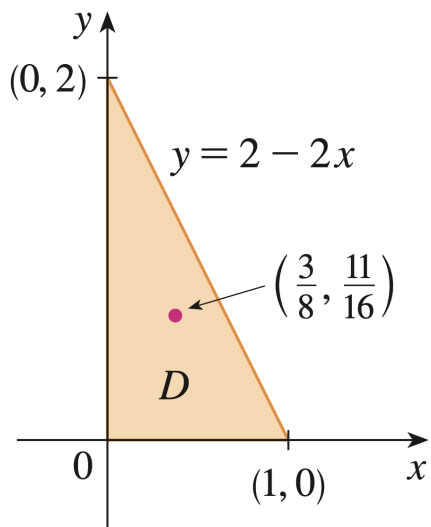
Definition. The moment of a particle about an axis measures the particle's tendency to rotate about that axis. It is computed by taking the product of the particle's mass and its distance from the axis. How can we compute the moment of an entire lamina about the x -axis and the y -axis?



Question. How can we compute the center of mass $(\bar{x}, \bar{y})$ of a lamina that occupies a region D and has density function $\rho(x, y)$?



Example. Find the mass and center of mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.



Example. The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

